ASAN REPORT

Exploring Possibilities: Minimum Number of Inspections Required to Detect a Violation in Denuclearization of North Korea

Kim Chong Woo, Ham Geon Hee

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The Asan Institute for Policy Studies

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About

The Asan Institute for Policy Studies is an independent, non-partisan think tank that undertakes policy-relevant research to foster domestic, regional, and international environments that promote peace and prosperity on the Korean Peninsula, East Asia, and the world-at-large.

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Executive Summary

Following the third inter-Korean summit in Pyongyang, there has been much discussion surrounding the possible denuclearization of North Korea. However, it remains to be seen if any tangible progress can be made towards *'final, fully-verified denuclearization.'* At the time of writing, it was difficult to predict even if a tentative agreement could be reached to allow nuclear inspectors into North Korea to examine and verify the denuclearization process. Against this backdrop, we have jumped ahead in time to focus our research on measuring the effectiveness of nuclear inspection in North Korea. In particular, we are interested in determining the probability of detecting violations assuming North Korea is not abiding by international law. The specifics of violation will be context-dependent. In our research, it was assumed from the outset that only a *'limited number'* of nuclear / WMD sites could be accessed by inspectors.

Essentially, we are interested in the probability of detecting at least one violation because one violation will constitute sufficient evidence that North Korea is again deceiving the international community. Our probability threshold is set at 90% throughout this research. But the choice of threshold is quite arbitrary. Our threshold has been carefully chosen to ensure that it is unlikely for a violation committed to go unnoticed. It is also assumed that each nuclear / WMD site is given an equal chance of committing a violation as it is difficult to collect intelligence on North Korea. This is the simplest assumption that can be made. We have provided the minimum number of inspections required to attain our threshold in each of examples used which include the Yongbyon nuclear complex, the critical and secondary nuclear / WMD sites and North Korea's underground facilities.

In Chapter 5, our findings are summarized in three tables. Using these tables, one can calculate the approximate minimum number of inspections required for a given number of sites, N. One of the tables is shown below.

It enables *'back of the envelope calculations'* to ensure that one can get an immediate sense of the magnitude of the task at hand. In the case of North Korea's Yongbyon nuclear complex, there are approximately ***400*** buildings (i.e., $N = 400$). A violation here can mean hiding fissile materials. Once it is assumed that there are ***5*** sites in violation (i.e., $n = 5$) among 400 sites (or buildings), then a minimum of ***164*** sites would need to be inspected

The number of inspections required for P as a percentage of N when $p = 0.9$[1]
(N used: 27~14,000)

	n				
P	2	3	4	5	10
90% (Threshold)	76.0~77.8 %	59.3~59.5 %	48.1~48.6 %	40.7~41.0 %	22.2~22.9 %
80%	61.4~63.0 %	46.2~48.1 %	36.8~37.0 %	29.6~30.6 %	14.8~16.5 %

to meet our threshold (i.e., $P = 90\%$).

In reality, it is highly unlikely for North Korea to risk committing a violation at the Yongbyon nuclear complex since it has indicated its willingness to permanently shut down the nuclear complex in return for corresponding actions from Washington as spelt out in the Pyongyang Joint Declaration. Therefore, it is necessary to carry out inspections at various sites beyond the Yongbyun nuclear complex.

According to the research conducted by Kristensen and Norris published in Bulletin of the Atomic Scientist, North Korea may have assembled 10~20 nuclear weapons. In terms of relevance to our study, suppose that there are 10 sites in violation. Here, a violation can mean hiding, on average, one or two nuclear weapons in each site corresponding to a total of 10 or 20 nuclear weapons respectively. From North Korea's perspective, there will be a trade-off between a need to minimize the risk of being detected by storing its nuclear weapons in fewer sites as possible and a need to store its nuclear weapons in multiple sites to minimize the risk of all of its nuclear weapons being detected. North Korea is unlikely to put all its eggs in one basket and moving these nuclear weapons will increase the probability of being detected while in transit.

Now, once it is assumed that these ***10*** sites (i.e., facilities) are among North Korea's ***14,000*** underground facilities (see section 4.4), then a minimum of approximately ***3,206*** sites would need to be inspected to meet our threshold.

1. The probability of correctly identifying a violation by the inspectors once they are at a violating site is denoted by p.

As the examples illustrate, this table is very useful in approximating the minimum number of inspections required. In all tables, the number of sites in violation, *n*, is deliberately assumed to be fairly low[2] (i.e., *n*: 2~5) while *P* is set quite high at both 90% (threshold) and 80% levels. Since this approach only provides a quick but approximate estimate on the minimum number of inspections required, one can always turn to Monte Carlo simulations to determine a more accurate estimate. The methodological details can be found in the appendix. Now, if no violation is found at the 90% threshold, then it will significantly reassure us that North Korea is abiding by international law / agreements. Of course, one needs to have a priori estimate on the number of sites in violation. This will be based on intelligence and an educated guess.

It must be borne in mind that we have only considered some aspects of inspection based on probability theory in this report. It goes without saying that good detective skills rooted in science and technology are required of inspectors to carry out their duties effectively.

1. Introduction

1.1 Background

In April 2018, South Korean President Moon Jae-in and North Korean leader Kim Jong-un held a summit at the truce village of Panmunjom within the demilitarized zone separating the two Koreas. A month and a half later, a historic summit between U.S. President Donald Trump and North Korean leader Kim Jong-un followed under the glare of the world's media on Singapore's Sentosa Island. The third inter-Korean summit in Pyongyang took place in September paving the way for the second summit between Trump and Kim. However, the Panmunjom Declaration, the Trump-Kim summit statement and a joint statement released after the third inter-Korean summit, failed to mention any concrete progress towards North Korea's denuclearization. In the document signed by Trump and Kim, North Korea reiterated its earlier positions indicated in the 1992 Joint Declaration on the Denuclearization of the Korean Peninsula, the 1994 US-DPRK Agreed Framework and the 2005 Six-Party Joint Statement.[3] North Korean violated all of its agreements. The results of this year's

summits fell short of our expectations. In particular, Trump's summit with Kim produced more noise than substance, but perhaps our expectations were too unrealistically high.

At the end of the third inter-Korean summit, North Korean leader Kim agreed to dismantle the Yongbyon nuclear complex but only if the United States takes reciprocal action. His conditional intentions could prove to be a stumbling block if a series of reciprocal actions eventually leads to Kim Jong-un demanding a U.S. troop withdrawal. Most South Koreans would strongly oppose this idea as U.S. military presence has been instrumental in defending South Korea from its communist neighbors for 65 years and brought great prosperity to the nation. Before the Trump-Kim summit, Kim Jong-un reportedly dropped his demand for U.S. troop withdrawal in exchange for denuclearization, but this was never an option on the table from the outset. So far the only real progress made is that he has agree to permanently dismantle the Tongchang-ri missile engine test site and a missile launch pad in the presence of international experts. This is a step in the right direction even though the site is of secondary importance. On the critical nuclear front, Kim Jong-un is likely to drag his heels as already observed. The U.S. intelligence assessment indicates that North Korea was producing enriched uranium even as it was engaged in diplomacy with the United States.[4] The U.S. Secretary of State Mike Pompeo once said that *'major'* North Korean nuclear disarmament can be satisfactorily achieved in two and a half years.[5] He has used a number of acronyms to succinctly describe the U.S. objectives. However, every time a new acronym appears, it sounds somewhat less provocative to North Korea than the one before. The following acronyms PVID (*'Permanent, Verifiable and Irreversible Dismantlement'*), CVID (*'Complete, Verifiable and Irreversible Dismantlement'*)[6] and FFVD (*'Final, Fully Verified Denuclearization'*) have all appeared over the period of several months to reflect a lack of progress and waning confidence. It remains to be seen if any real progress towards

2. This is relative to the total number of sites, *N*.
3. Michael J. Green, "The Trump-Kim Summit: Outcomes and Oversight," Statement before the House Committee on Foreign Affairs Subcommittee on Asia and the Pacific, 20th June 2018.
4. "North Korea has increased nuclear production at secret sites, say U.S. officials," NBC News, 1st July 2018.
5. "North Korea Nuclear Deal Could Take '*Years*,' Trump suggests," The New York Times, 26th September 2018. It appears that Trump has relaxed his timeline for North Korea's denuclearization.
6. Heather Nauert, Department Press Briefing, U.S. Department of State, 3rd May 2018.

North Korea's denuclearization / dismantlement can be made following the third inter-Korean summit. It is worth mentioning how Thae Yong-ho, former North Korean Deputy Ambassador to the United Kingdom, sees the future unfolding. He claims that North Korea will never agree to end its nuclear weapons programs through a *'Complete, Verifiable and Irreversible Denuclearization,'* partly because besides nuclear and missile facilities, it has many sensitive places such as political prison camps which can never be open to the outside world for inspection.

1.2 Inspection and Verification

Against this backdrop, we have jumped ahead in time to do research relating to inspection and verification. *'Complete'* denuclearization necessitates proper inspection and verification. This can only begin once North Korea has implemented *'complete'* denuclearization and is ready to go through a stringent inspection and verification process. When this process has been satisfactorily completed and North Korea has returned to the Nuclear Nonproliferation Treaty as a non-nuclear weapon state, the international community can begin lifting sanctions. Even then verifying *'complete'* denuclearization in the literal sense of a word is not attainable. The only way of achieving this would be to search the whole of North Korea with a fine-tooth comb; therefore, in that regard, the term *'complete'* is a misnomer. Perhaps adopting a *'degree of assurance'* would be better suited for the purpose. One of the most important functions of an inspection system is to provide this assurance that North Korea is keeping its agreements. However, it is virtually impossible to discard the possibility that some nuclear warheads are hidden.

At the time of writing this report, it was difficult to predict even if a tentative agreement could be reached to allow nuclear inspectors into North Korea to begin the examination and verification process. If such a time ever comes, it is essential to have some means of measuring the effectiveness of inspection. We are interested in determining what our chances are of detecting any violation assuming North Korea is cheating. More specifically, how many inspections are required and how efficient should the inspectors be in detecting a violation to increase our overall chances of detection to 80% or 90%. The specifics of violation will be context-dependent. With the aid of a mathematical model, these questions can be answered. The model considers the situation in which only a limited number of nuclear sites in North Korea can be accessed *simultaneously* by the inspectors. This is assumed on the grounds that North Korean authorities are unlikely to give the inspectors total freedom to check on any number of nuclear sites or that the IAEA does not have enough manpower necessary to carry out thorough inspections. Each nuclear site is given an equal chance of committing a violation as it is difficult to collect intelligence on North Korea. The inspectors and North Korean authorities will be engaged in a game of cat and mouse trying to outguess each other, another reason for assigning the equal chance. This is the simplest assumption we can make. These limited nuclear sites are randomly chosen and if particular sites are suspected with *'good reason'* to have committed violations (e.g., there might be an evidence), then the overall chances of detecting any violation should be greater than the one based on our calculations. Put simply, our calculations can provide baseline estimates. It is noted that the inspection process normally begins only after receiving a list of nuclear sites declared by North Korea.[7] The inspectors can use their expert knowledge to find promising leads in the search for violations.

2. North Korea's Nuclear / WMD facilities

One of the factors (i.e., parameters, variables or inputs in mathematics) that determines the model output is the number of nuclear / WMD sites in North Korea. Any number of these sites can be chosen and visited by the inspectors, but not all of them by assumption. In our research, the following sources have provided information regarding the number of nuclear / WMD sites. A 2014 RAND report by T. Bonds, E. Larson, D. Eaton and R. Darilek discusses and lists 141 North Korean WMD sites in detail.[8] The WMD sites include not only nuclear sites but also biological, chemical weapons and missile sites. Also, Joseph Bermudez, the author of three books on the North Korean military, thinks North Korea has 11,000-14,000 underground facilities. Stockpiles of biological, chemical or nuclear weapons could be easily hidden in some of these facilities, and they could even house a production line for producing highly-enriched uranium. The Chosun Ilbo[9] also reported that there are some 390 buildings in North Korea's Yongbyon

7 It is quite another matter whether it will provide a full list.

8. Timothy Bonds, Eric Larson, Derek Eaton, Richard Darilek, "Strategy-Policy Mismatch, How the U.S. Army Can Help Close Gaps in Countering Weapons of Mass Destruction," RAND report, 2014.

nuclear complex while the New York Times[10] put this number at 663 buildings citing the Institute for Science and International Security as the source. Some of these numbers have provided input to the model.

Earlier, we mentioned Thae Yong-ho's thoughts on North Korea's political prison camps as being no-go areas for the inspectors. Besides these prison camps, such places as Kim Jong-un's holiday homes are considered to be even more inaccessible and sensitive. Some of them can be used to hide nuclear assets (e.g., warheads, centrifuges). However, counter-intuitively, these holiday homes can be opened up to the foreign media as evidence to support North Korea's compliance. It will score a great propaganda coup. Meanwhile, the political prison camps can never be opened up irrespective of whether they are housing nuclear assets or not, so there is a qualitative difference between the two potential storage places. North Korea can insist both these places are off-limits for nuclear inspections, but at the same time these places are targets of interest for spy satellites. Any suspicious activities can be detected by these satellites rendering them easy target should military action be taken against them.[11] For North Korea, storing of its nuclear assets in total safety and secrecy is of paramount importance. There are thousands of underground facilities available which would better suit this purpose.

3. The Descriptions of Mathematical Models (Analytical & Monte Carlo)

In our previously published work,[12] our overall chances of detecting any violation, as described in Section 1.2, were calculated using a mathematical formula (see A1). It

9. "What are N.Korea's Nuclear Facilities?" The Chosun Ilbo, 5th May 2018.
10. "The Nine Steps Required to Really Disarm North Korea," The New York Times, 11th June 2018.
11. It could be argued that North Korea can use political prisoners as human shields to ward off a U.S. strike. However, the urgency to destroy North Korea's nuclear capability would take precedence over anything else.
12. Kim Chong Woo and Ham Geon Hee, "Quantifying the Effectiveness of Nuclear Inspection in North Korea," Asan Issue Brief, 20th July 2018.

was an analytical technique that provided an answer to this question. The R statistical package was used to turn the mathematical formula into code. We are interested in determining P, the probability of detecting at least one violation assuming that North Korea is cheating. A higher value of P means an increased chance of catching North Koreans red-handed. P is determined by five factors (i.e., $P(M, N, n, m, p)$). The minimum number of violations we are interested in detecting is denoted by M. Throughout our research, we have only considered *'at least one or more violations'* rather than, for instance, at least two or more violations. One violation will constitute sufficient evidence that North Korea is again deceiving the international community. The (total) number of nuclear / WMD sites is denoted by N. Every one of these sites is a potential target for inspection. If there is a site unknown to the inspectors (i.e., not included in N), then this lies outside of the scope of the model. Hence, it is important to have as complete a list of targets as possible. If North Korea categorically refuses to allow inspection of a particular site, then this site is also not included in N and our equation for P cannot be applied. Depending on the context, N can refer to a number of buildings, facilities or some other structures. Next, the number of sites in violation among N is denoted by n. North Korea will try to hide these sites from the inspectors, and do its utmost best not to leave any incriminating evidence behind at these sites. Typical examples of violations include hiding nuclear warheads, fissile materials, precision tools and equipment needed to manufacture nuclear weapons. In reality, the value of n will be based on intelligence and an educated guess. In our research, ranges of values are considered. If one range is favored over the other, there must be a reason as it has direct implications on a number of inspections required. The number of inspections allowed is given by m, and m sites are randomly chosen. Finally, the probability of correctly identifying a violation by the inspectors once they are at a violating site is denoted by p. The inspectors must not fail to miss any incriminating evidence once they are at the site in violation. It is important to have a higher p value.[13] There should be no confusion between the big 'P' and the small 'p' as their meanings are clear. The mathematical equation for P and the conditions under which it can be used for calculations are explained in detail in A1.

The mathematical equation for P is very useful, but it has its limitations. It can only determine the value of P as long as the number of inspections allowed (i.e., m) is no

13. This is not to be confused with a P-value in hypothesis testing.

greater than the number of sites in violation (i.e., n). So, for instance, if North Korea is hiding its nuclear warheads in 10 sites (i.e., $n = 10$) among 100 sites (i.e., $N = 100$), then this equation can only calculate the value of P in which the number of inspections allowed is at most 10 (i.e., $m \leq 10$). In other words, if inspections are to take place at 11 sites, the value of P cannot be determined. This analytical technique is rather restrictive in its scope as P cannot be readily obtained. However, there is another mathematical technique that we can utilize to overcome this restriction. It is the Monte Carlo Technique which can free us from this undesirable constraint on the number of inspections allowed (i.e., m). This statistical technique basically harnesses the computing power to approximately determine the value of P. As the number of iterations increases, the approximate value comes closer and close to its true (analytical) value. It is found that 1,000 iterations (samplings) are sufficient to bring the value of P to be identical up to two decimal places.[14] Throughout this report, all values of P were determined using this technique unless stated otherwise. Its methodological details can be found in A2.

4. Simulation Findings

4.1 Yongbyon Nuclear Complex

Our previous work focused on North Korea's Yongbyon nuclear complex with its approximate 400 buildings substituted in for the number of nuclear sites, N (i.e., $N = 400$). Yongbyon was chosen to illustrate how P evolves with different values of n, m and p.[15] Our research has expanded on this earlier work with the use of Monte Carlo Technique. This technique has allowed us to calculate the minimum number of inspections required to attain a 90 % probability of detecting at least one violation committed by North Korea under specific circumstances. Here, for example, a violation could mean hiding

14. Time was also another factor in limiting the number of iterations to 1,000. The additional time spent on running 10,000 iterations to gain further accuracy in P value would not have been worthwhile.

15. In fact, North Koreans would be foolish to leave any evidence of violation in such a well-known nuclear complex like Yongbyon.

fissile materials. If North Korea is found not to be in violation even after allowing this number of inspections, then, it will significantly reassure us.

Figure 1. Probability of detecting at least one violation (Analytical Technique)

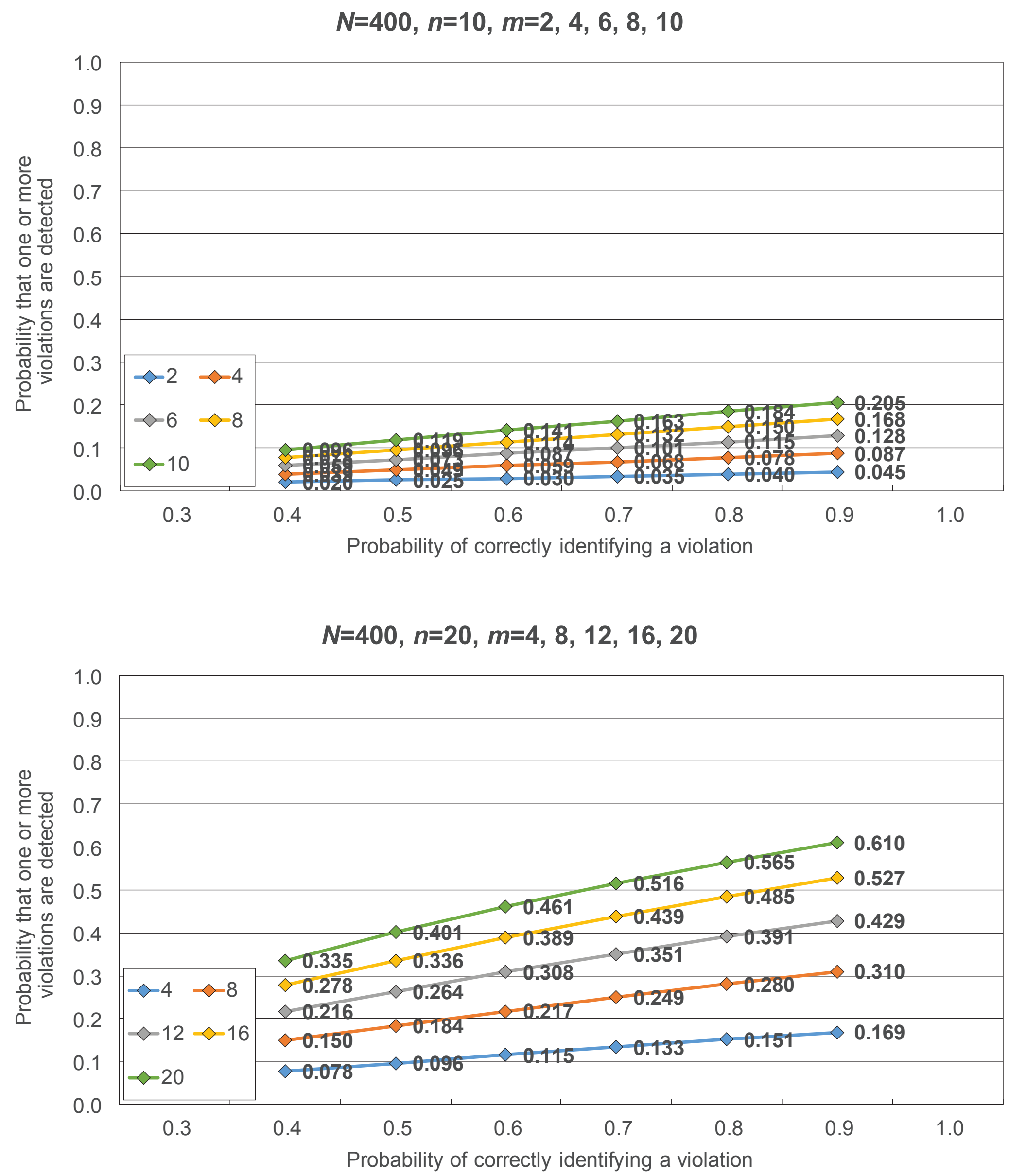


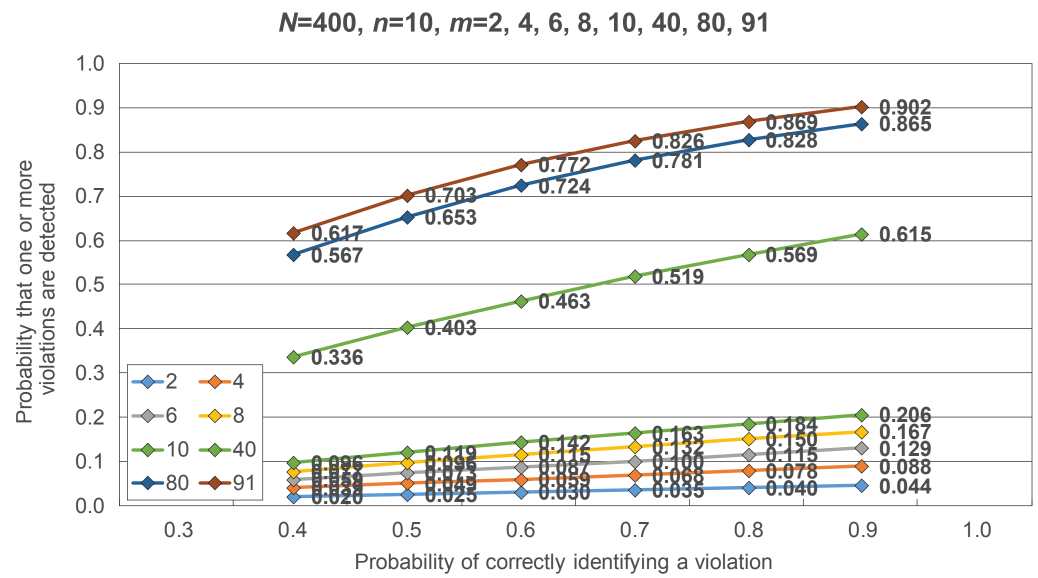
Figure 1 shows *P*, the probability of detecting at least one violation (i.e., one or more violations), on the *y*-axis. The small *p*, the probability of correctly identifying a violation, is represented on the *x*-axis. Here, all the values of *P* are calculated using the analytical technique for $n = 10$ and $n = 20$. Consider first the top chart with 10 sites in violation at the Yongbyon nuclear complex. For a pair with a given *m* ranging from 2 to 10 and a given *p* ranging from 0.4 to 0.9, its corresponding *P* value marked by a diamond-shaped symbol is given. Here, both *P* and *p* have the maximum value of 1.0 denoting 100%. So, for instance, a *p* value of 0.4 means a 40% probability.

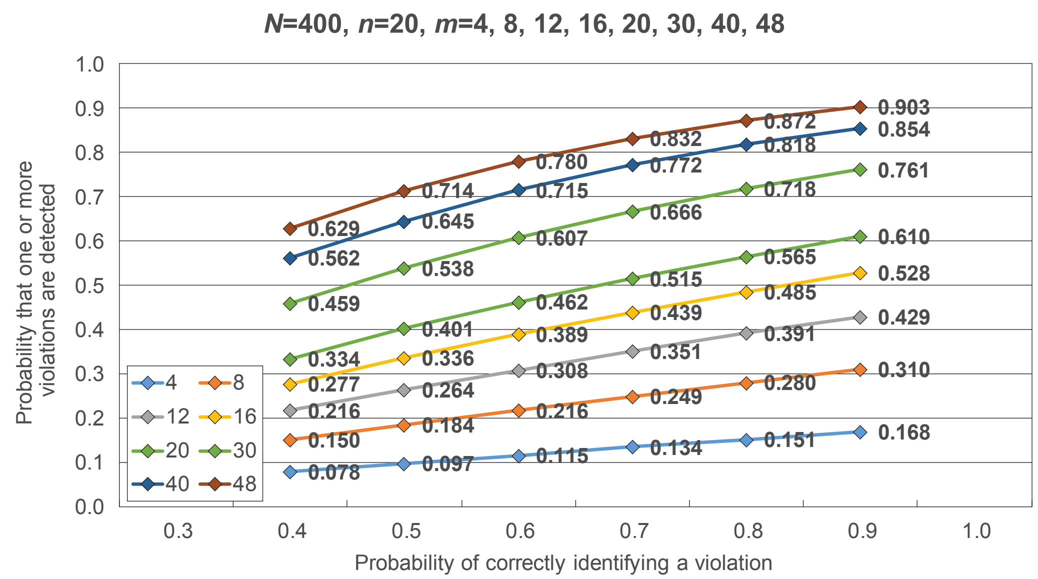
For the worst performing pair ($m = 2, p = 0.4$), its *P* value is only 0.02. In plain English, this means that when North Korea is hiding some of its nuclear assets in 10 different buildings among 400 buildings at Yongbyon and the inspectors can only choose to visit 2 places with a 40% chance of correctly identifying each of these places to be in violation if it happens to be, then the overall chance of detecting at least one violation is only 2%. For the pair (10, 0.9), its *P* value improves to 20.5% which is still very low. This is as far as the analytical technique can go as the number of inspections, *m*, cannot exceed 10 (i.e., $\leq n$). Consider the bottom chart with the number of sites in violation doubled to 20. Based on *n*, *m* is proportionally worked out as in the previous case with its range from 4 to 20. Each of the pairs (4, 0.4) and (20, 0.9) give its *P* value of 7.8% and 61% respectively. The latter is not sufficient for assessing North Korea's commitment to denuclearization. We need to be able to determine *P* beyond 20 inspections. The threshold is set at attaining a 90 % probability of detecting at least one violation. So, the question is, *'how many number of inspections are required to meet this threshold?'*

In Figure 2 below, all values of *P* are calculated using the Monte Carlo Technique for $n = 10$ and $n = 20$. Each chart has all the values of *m* tested in Figure 1 as well as some new ones. Each *P* value is the result of running 1,000 iterations. One can observe that a *P* value is identical to two decimal places compared to its analytical counterpart in Figure 1.[16] For $n = 10$, it is calculated that inspecting 91 sites is the minimum number of inspections required to obtain a *P* value of 90.2% with *p* set at 0.9. It is not certain what a realistic value of *p* is on average, but its desirable features will include predictability and consistency (i.e., a low standard deviation). Any value less than 91 for *m* and 0.9 for *p* will not meet the threshold. For $n = 20$, a *P* value of 90.3% can be obtained with

16. This means it is identical up to the integer part of a real number in percentage terms.

Figure 2. Probability of detecting at least one violation (Monte Carlo Technique)

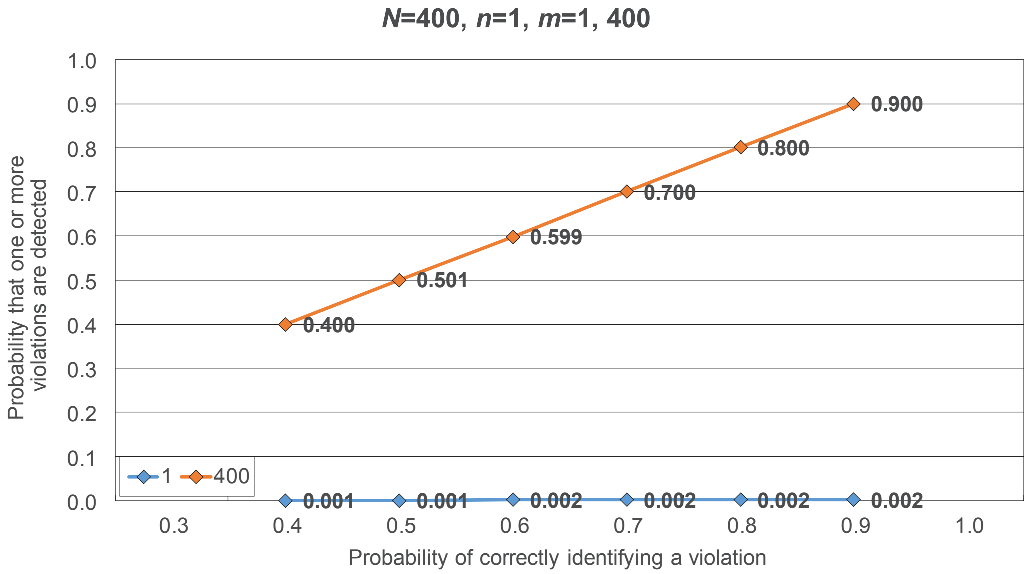


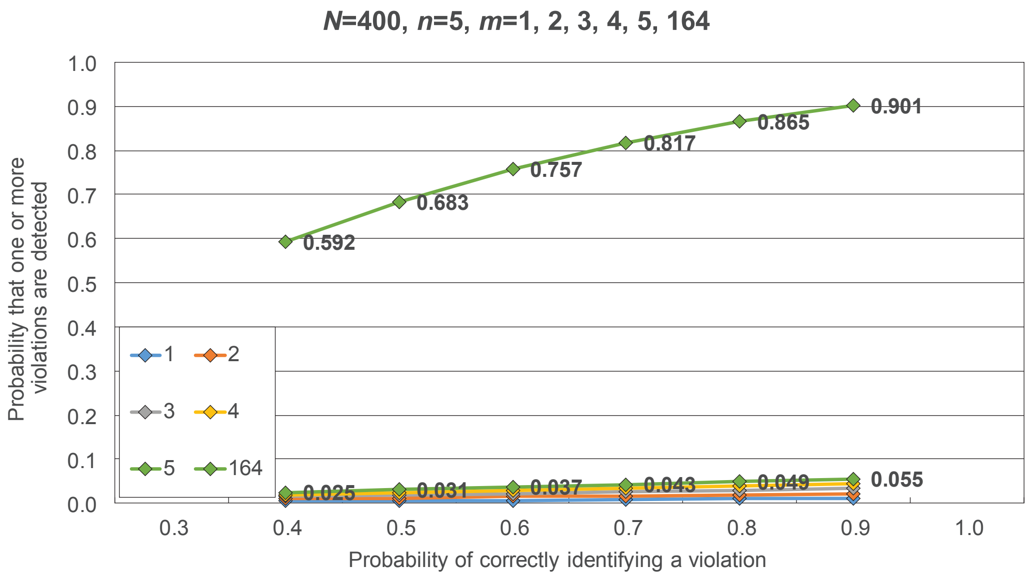


the minimum of 48 inspections. This meets the threshold required with *p* at 0.9. As can be observed, there are more than one way of obtaining a given value of *P*. For example, each of the pairs (48, 0.5) and (40, 0.6) give its respective *P* value of 71.4% and 71.5%.

The values are similar although m and p are different. This illustrates how m and p can be traded-off while retaining P. For low values of P, it appears that a value at $p = 0.9$ is approximately double that of $p = 0.4$ for a fixed value of m. This doubling tendency

Figure 3. Probability of detecting at least one violation when *n* is very small





diminishes when the number of inspections, m, is large.

Consider the case when the number of sites in violation is very small. Figure 3 shows several values of P when there is either only 1 or 5 sites in violation among 400 sites. Assume that North Korea has hidden its nuclear warheads in a single site, what does P tell us? If the inspectors are given just one chance, then there is a 0.2% probability of this site being detected with p fixed at 0.9. Intuitively, there is a 90% probability of this site being detected after inspecting all 400 sites when p is 0.9. This stems from the fact that the probability of correctly identifying a violation, p, at this site is 0.9. The non-violating sites contribute nothing to P. However, when p falls below 0.9, it is not possible to meet this threshold. Consider 5 sites are in violation. 164 inspections will result in a P value of 90.1% meeting the threshold. This confirms common sense that a large number of inspections are required to detect a small number of sites in violation.

When it comes to hiding its nuclear warheads, North Korea is unlikely to put all its eggs in one basket. There will be a trade-off between a need to minimize the risk of being detected by storing its warheads in fewer sites as possible and a need to store its warheads in multiple sites to minimize the risk of all of its warheads being detected. When faced with this problem, how will North Korea trade-off?

4.2 Nuclear Sites (RAND estimates)

In this section the number of nuclear sites (or facilities) has been narrowed down to include only those that are of significant importance. The 2014 RAND report mentioned earlier in Chapter 2 has specifically identified 39 nuclear sites that need to be secured or searched in the event North Korea collapses. The authors of the report have analyzed the NTI (Nuclear Threat Initiative) data on North Korea's WMD and missile sites.[17] We draw upon their findings here. The 39 sites are grouped into 9 *'Critical Sites'* and 30 *'Secondary Sites.'* One of the critical sites is actually 19 facilities within the confines of the Yongbyon Nuclear Research Center, but treated as one. Therefore, there are 57 sites in which 27 are critical sites and 30 are secondary sites. Their list of North Korea's nuclear sites grouped by priority is shown below in Table 1.

17. Nuclear Threat Initiative, http://www.nti.org/learn/countries/north-korea/.

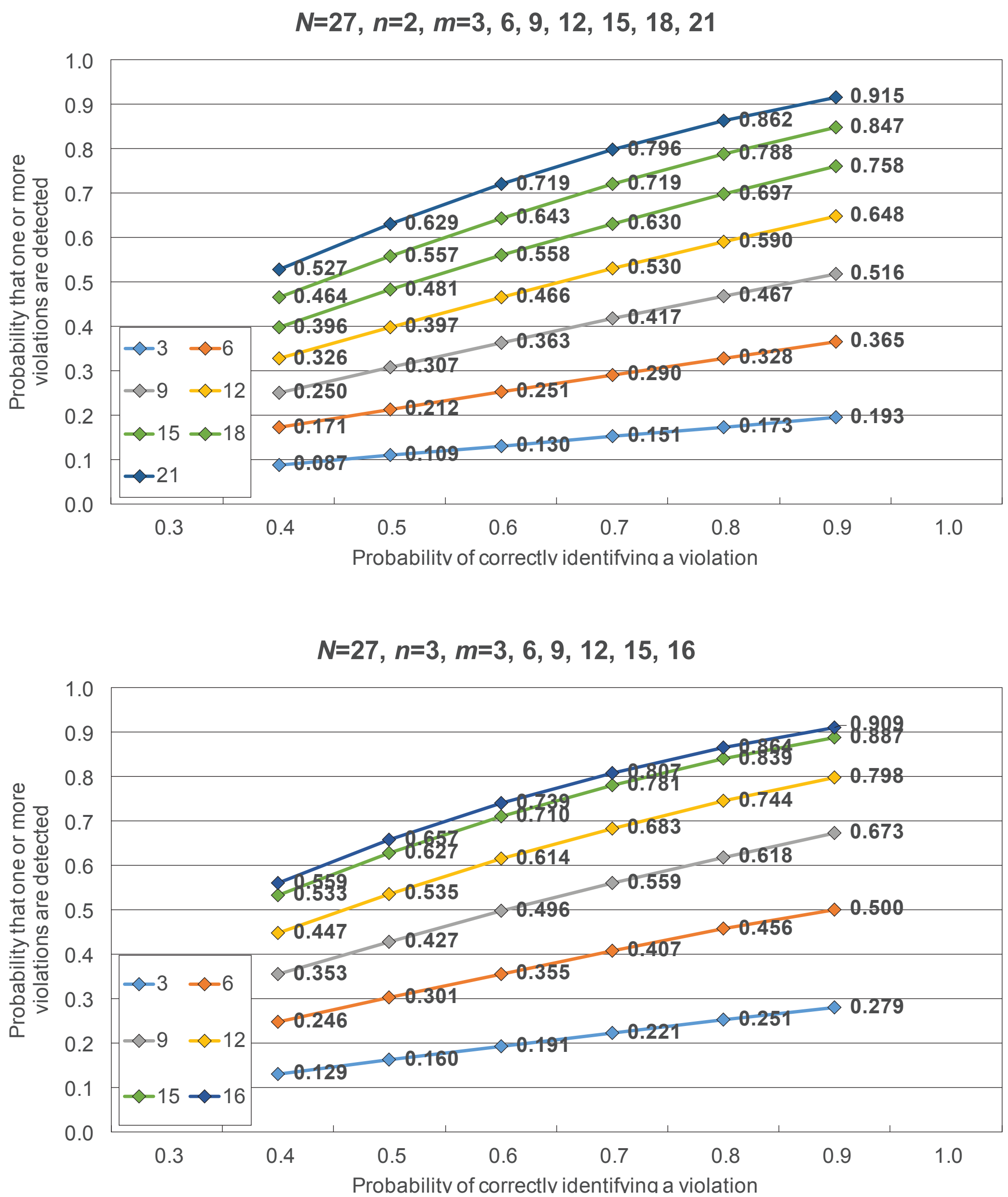
Table 1. NTI Nuclear Facilities in North Korea

Critical Sites	
Yongbyon Nuclear Research Center (1) • 19 of NTI's nuclear facilities are located in the Yongbyon Nuclear Research Center **Nuclear Storage Sites (1)** • Geumchang-ri Underground Facility **Nuclear Test Site (1)** • Punggye-ri Nuclear Test Facility **Nuclear Weaponization Facilities (1)** • Yongdeok-dong High-Explosive Test Site	**Nuclear Enrichment Facilities (5)** • Taecheon Underground Suspected Nuclear Facility (Enrichment & Reprocessing) • Bakcheon Underground Nuclear Facility • Suspected Cheonmasan Uranium Enrichment Facility • Hagap Underground Suspected Nuclear Facility • Yeongjeo-ri Suspected Uranium Enrichment Facility
Secondary Sites	
Nuclear Research and Development (5) • Kim Chaek University of Technology (Pyongyang) • Kim Il Sung University (Pyongyang) • Korea National Defense College (Kanggye) • Laser Research Institute (Pyongyang) • MGC-20 Cyclotron (Pyongyang) **Nuclear Education and Training (2)** • P'yŏngsŏng College of Science • Hamheung University of Chemical Industry Unfinished Nuclear Power Reactors (2) • Geumho-Jigu Light Water Reactor Site (never built) • Taecheon 200MWe Nuclear Reactor (never finished)	**Facilities Associated with Uranium Mining and Processing** **Nuclear Milling (5)** • Bakcheon Uranium Milling Facility • Cheonmasan Uranium Milling Facility • Korea International Chemical Joint Venture Company • Kusong Uranium Milling Facility • P'yŏngsan Uranium Milling Facility **Nuclear Mines(16)** • Ch'ŏlsan Uranium Mine • Haegumgang Uranium Deposit • Hamhung Uranium Deposit • Hwangsan January Industrial Mine • Hyesan Uranium Mine • Hungnam Uranium Mine • Kujang Uranium Mine • Kumchon Uranium Mine • Musan Uranium Mine • Najin Uranium Mine • P'yŏngsan Uranium Mine • Pakch'ŏn Uranium Mine • Shinp'o Uranium Mine • Sunch'ŏn Uranium Mine • Sŏnbong Uranium Mine • Wiwŏn Uranium Deposit

Source: Timothy Bonds, Eric Larson, Derek Eaton, Richard Darilek, "Strategy-Policy Mismatch, How the U.S. Army Can Help Close Gaps in Countering Weapons of Mass Destruction," RAND report, 2014.

All of these nuclear sites should be thoroughly investigated by the inspectors if and when the process begins. But the focus of our research has been on finding P under the assumption that only a limited number of these sites can be accessed by the inspectors.

Figure 4. Probability of detecting at least one violation for the critical nuclear sites, N=27



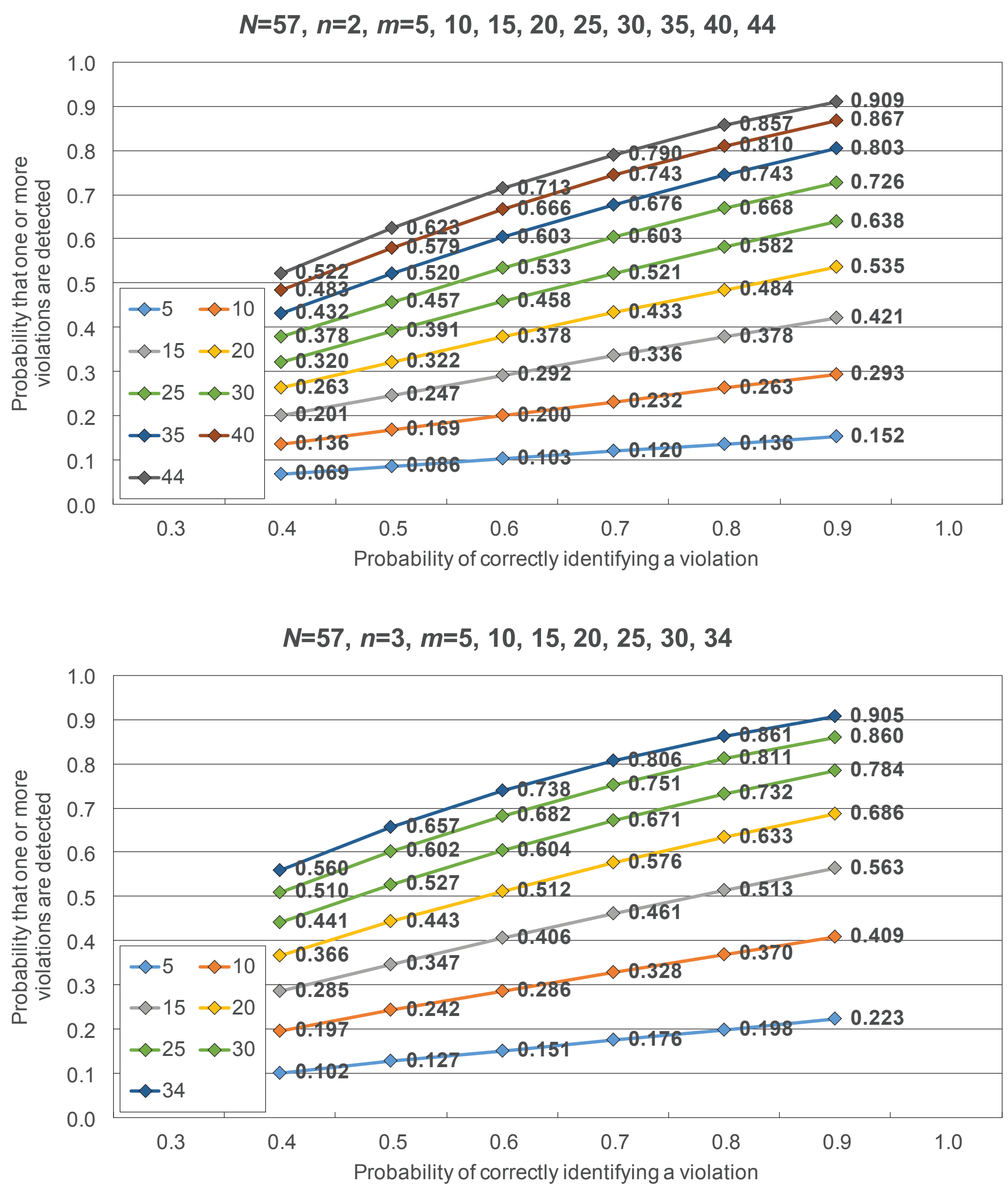
We have considered two cases, one with the 27 critical sites, and the other with the 57 nuclear sites consisting of both the critical and secondary sites (i.e., N = 27 and 57).

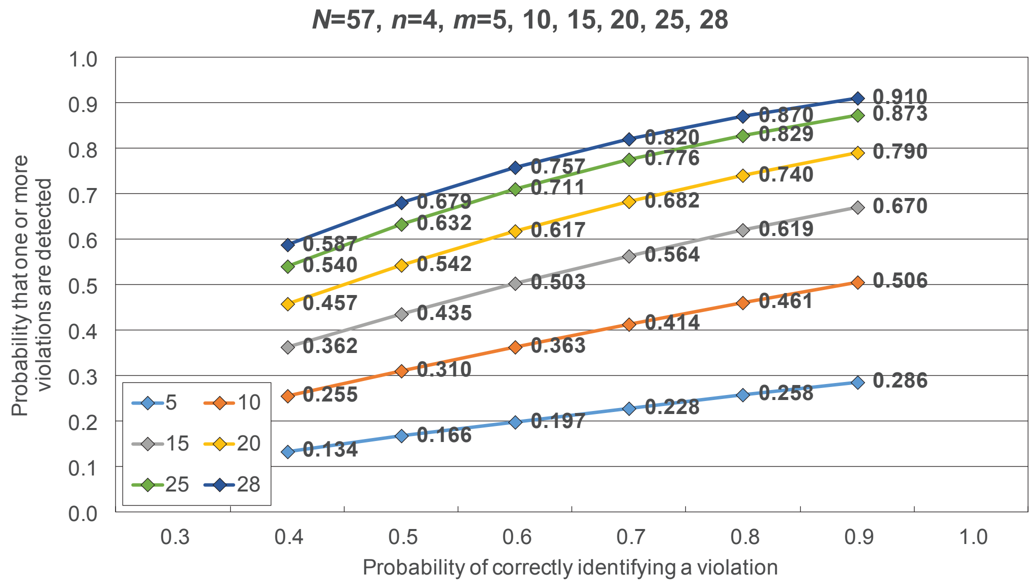
Figure 4 shows two charts for the critical sites. The top chart shows P when only 2 sites among the 27 sites are in violation. The number of sites in violation is very low here (i.e., n = 2), and it requires at least 21 inspections for P to attain a value of 91.5% crossing the threshold. The small p is held at 0.9. If p slides below 0.9, then additional inspections are needed to keep P above the threshold. It is observed that even when p is as low as 0.4, the corresponding value of P is 52.7% which is still above 50%. The bottom chart shows P when one more site is in violation (i.e., n = 3). The number of inspections required for P to meet the threshold decreases to 16 inspections which is 5 less than the previous case. The attained value of P is ~91% with p fixed at 0.9. It is possible to directly compare the corresponding values of P between the two charts as only n differs while other factors are held constant. For example, when n = 2, P has a value of 75.8% with 15 inspections and p at 0.9. But P increases to 88.7% under identical conditions except n = 3. There is a 12.9% point increase. Should North Korea have only *one site* in violation, then all 27 sites would have to be inspected in order to meet the threshold. This is applicable to all other values of N. In essence, with only a single site among N in violation, searching every known nuclear sites (i.e., N) will always give P the 90% threshold as long as p remains 0.9. It follows from this result that P is equal to p for any value of p within its range.

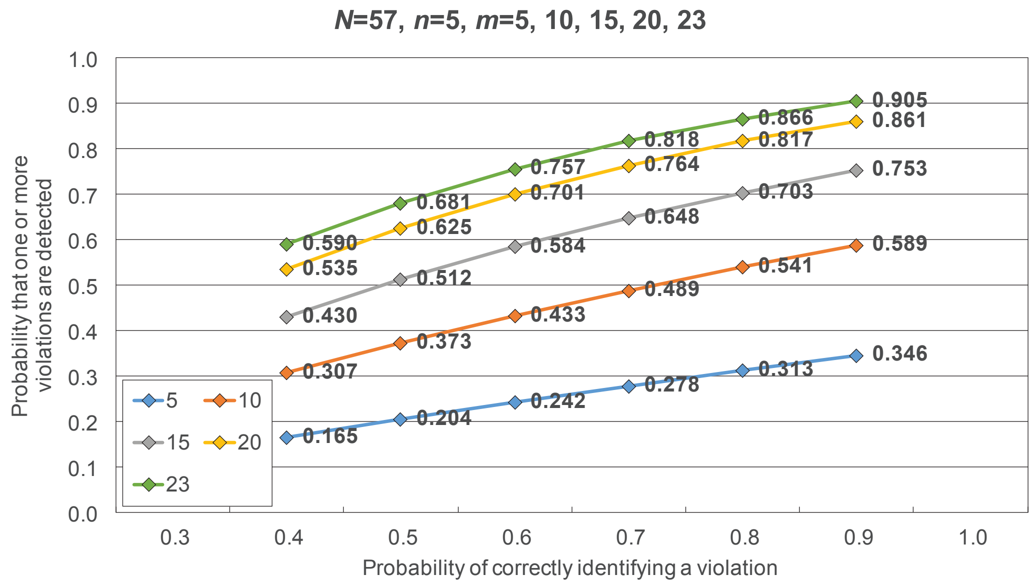
Let us now consider all 57 nuclear sites. These include 30 secondary sites consisting of 21 uranium mining and processing sites, 2 unfinished nuclear power reactor sites and 7 sites associated with nuclear research and development or educational training. Figure 5 below has four charts, each showing P with a fixed number of sites in violation. This fixed number, n, takes a value of 2, 3, 4 and 5. The number of nuclear sites, N, is 57. Once again, the question is, what are the minimum number of inspections required to meet the threshold at P = 90%? For each n = 2, 3, 4 and 5, the required numbers are 44, 34, 28 and 23 inspections respectively while p is fixed at 0.9. In each chart, the last point on the top curve shows P hovering between 90 and 91%. It is observed among the top curves that the value of P decreases more steeply with decreasing values of p when the number of sites in violation, n, is smaller. For instance, a drop in the value of P going from p = 0.9 to 0.8 is 0.052 when n = 2, but when n = 3, it is 0.044. Consider the case n = 2 with 44 inspections and p fixed at 0.4. Its corresponding P is 52.2%. When p increases to 0.9, P is 53.5% with only 20 inspections. Both values of P are in the vicinity

of each other, yet we need 24 inspections less when p is 0.9. This makes it all the more important to have a high value of p.

Figure 5. Probability of detecting at least one violation for the critical and secondary nuclear sites, N=57







It is noted that in selecting inspections sites, m, all 57 sites are treated equally. There was no discrimination between the critical and secondary sites. In the case of n = 5, 23 inspections would suffice to meet the threshold even though there were 27 critical sites.

4.3 WMD Sites (RAND estimates)

In this section, we considered all WMD (Weapons of Mass Destruction) sites which include nuclear, chemical and biological weapons sites. It also includes North Korea's ballistic missile sites. Table 2 shows the number of critical and total sites grouped by site category. It is based on the 2014 RAND report which has a detailed analysis of these WMD sites as part of a study into WMD-Elimination (WMD-E) operations. For chemical weapons, the report lists 6 storage facilities, 5 chemical weapons production facilities, 15 dual-use facilities and 12 other chemical weapons associated facilities and organizations. Out of 38 potential sites, 15 sites are identified as critical (high-priority) sites. For biological weapons, there are 1 potential weaponization facility, 3 production facilities and 11 potential research and design facilities and organizations. Out of 15 potential sites, 10 are identified as critical sites. For North Korea's ballistic missile sites, there are 25 missile bases, 22 missile production facilities and 2 research and design facilities. Among these sites, 12 are identified to be critical sites.

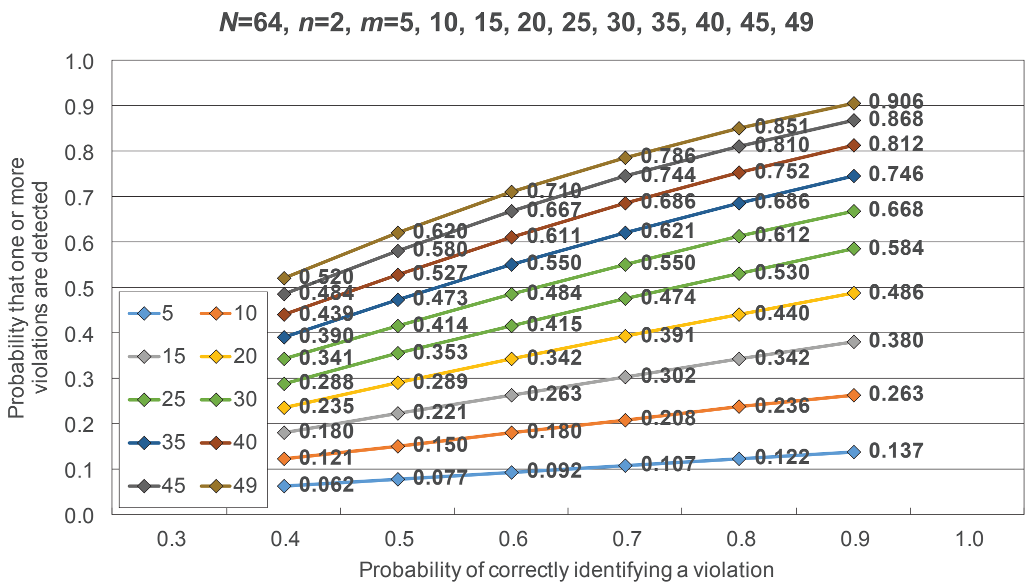
Table 2. The number of critical sites and sites in total grouped by site category

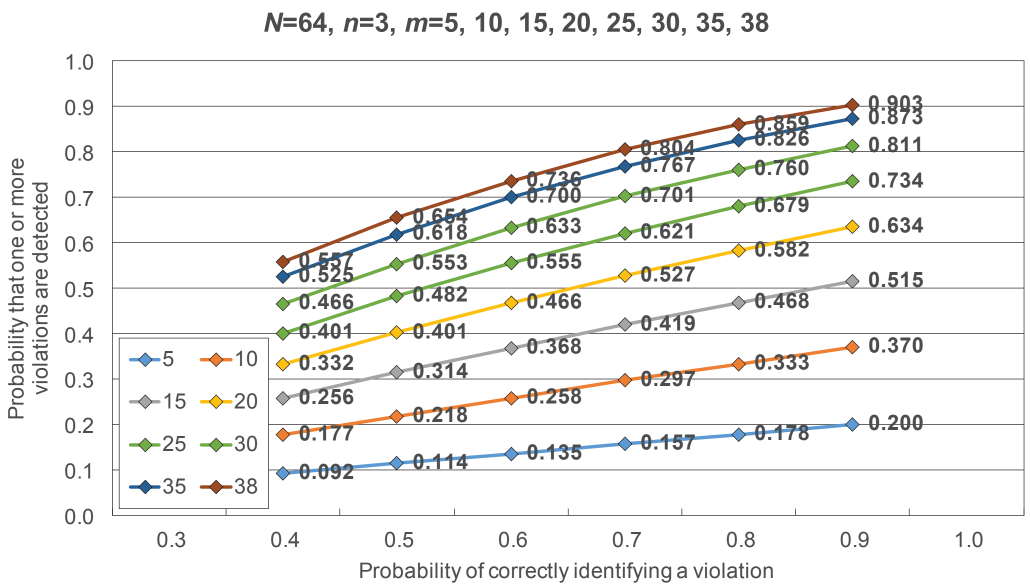
Site Category	Critical Sites	Total Sites
Nuclear	27	57
Chemical	15	38
Biological	10	15
Missile	12	49
Total	64	159

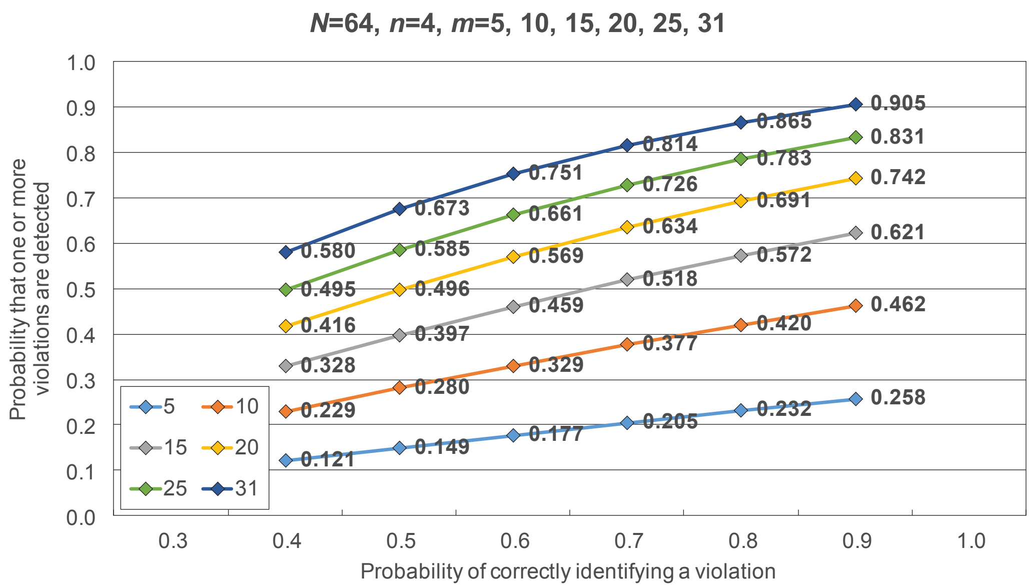
Source: Timothy Bonds, Eric Larson, Derek Eaton, Richard Darilek, "Strategy-Policy Mismatch, How the U.S. Army Can Help Close Gaps in Countering Weapons of Mass Destruction," RAND report, 2014.

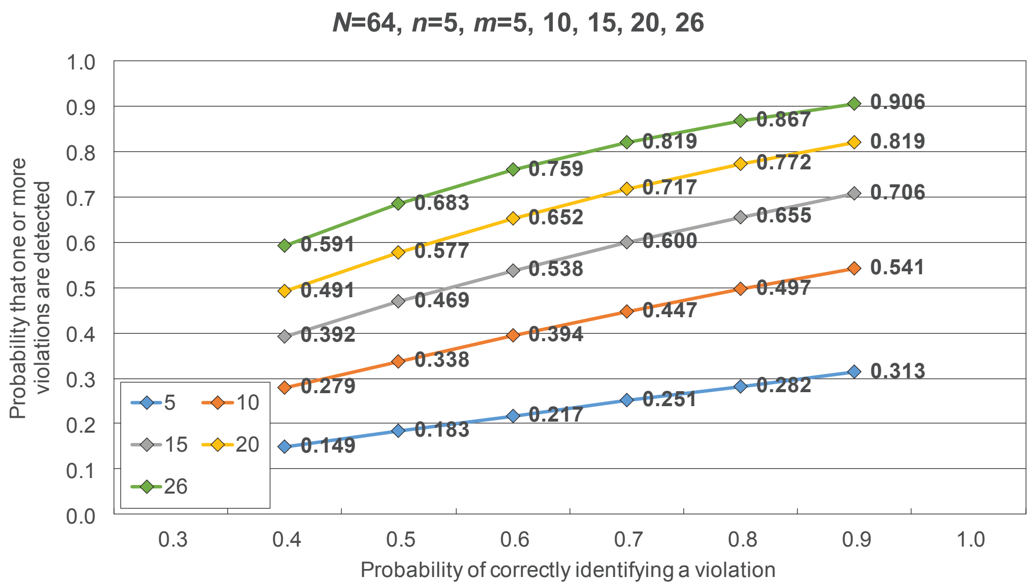
In total, there are 64 critical WMD sites out of 159 sites which also include secondary sites. Consider first only the critical WMD sites. Figure 6 below illustrates how P evolves when the number of sites in violation takes values (i.e. n) 2, 3, 4 and 5 as before. To meet the threshold, 49, 38, 31 and 26 inspections need to take place respectively. For n = 2, inspecting 49 sites among the 64 sites will ensure P to attain 90.6% with p fixed at 0.9. This means that 76.5% of N sites need to be searched. It is also observed that, in the previous case, 44 sites were needed among the 57 sites to achieve the threshold with the same n and p. Here, 77.2% of N sites need searching. For n = 3, 38 out of the 64 sites corresponds to 59.3%. In the previous case, the number obtained under similar circumstances was 34 out of the 57 sites. This corresponds to 59.6% of *N*.

Figure 6. Probability of detecting at least one violation for the critical WMD sites, *N*=64





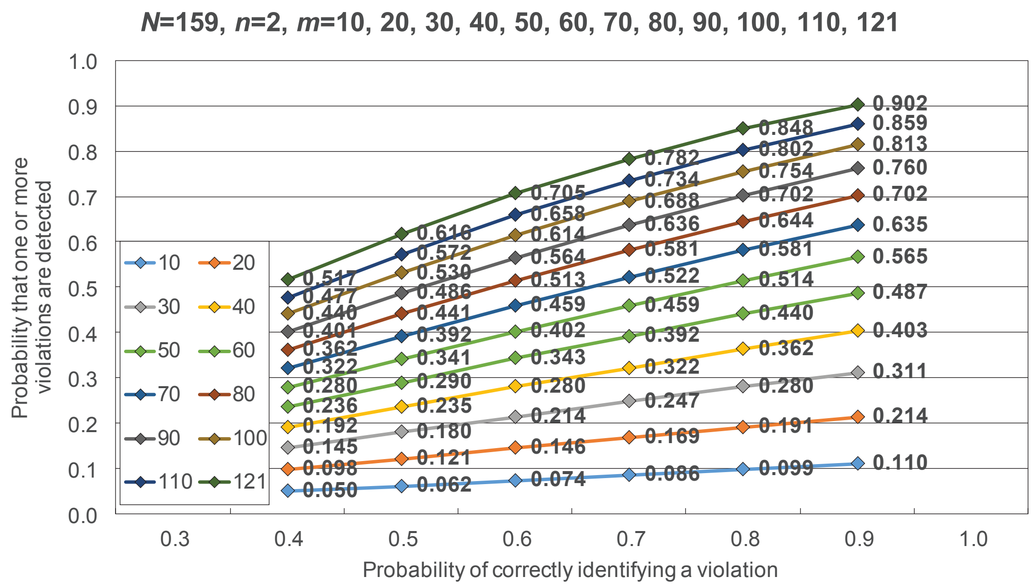


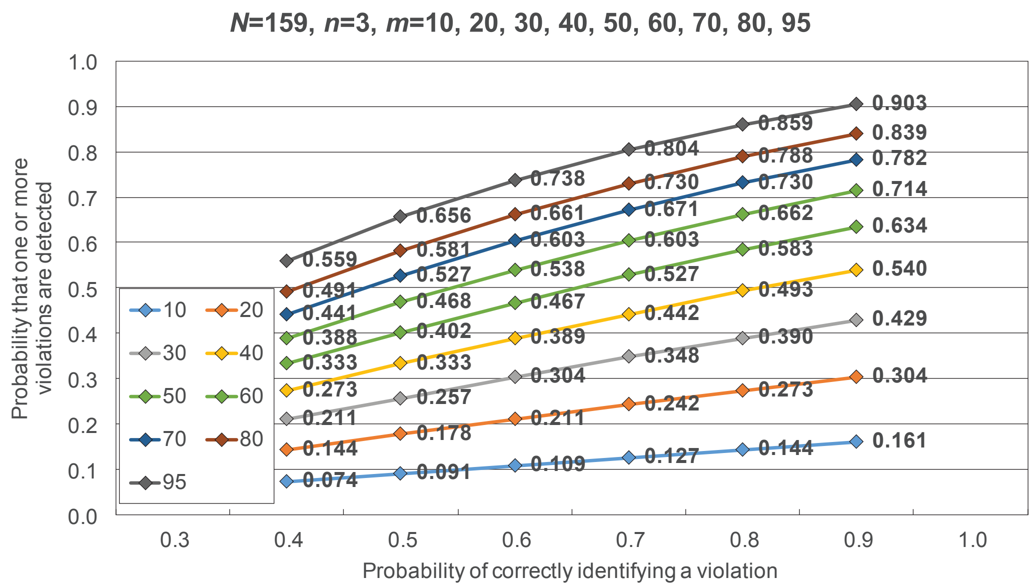


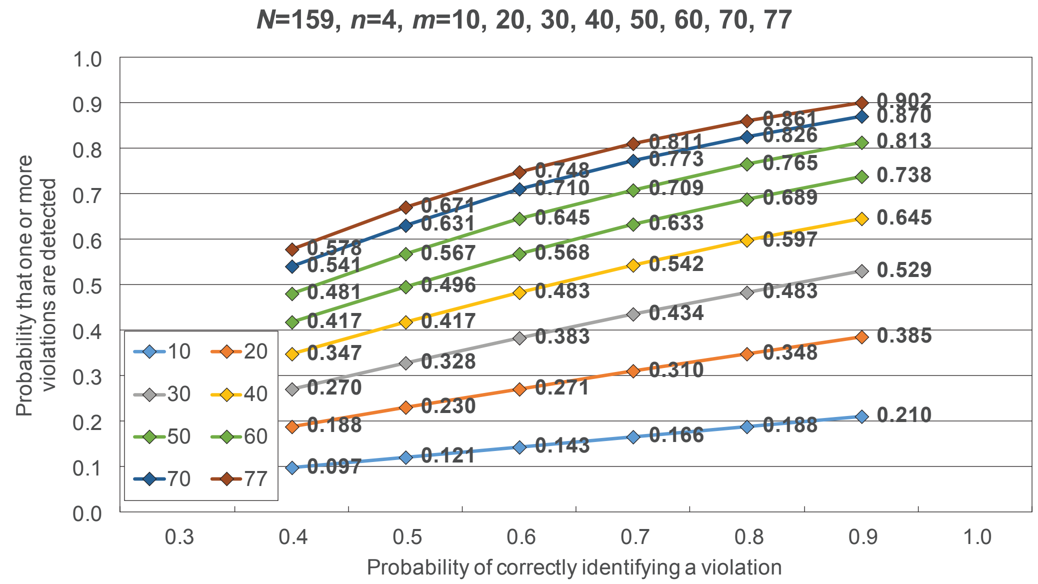
It can be inferred that the minimum number of inspections can be approximately estimated as a fixed percentage of *N* for given *n* and *p* (*P* is already fixed at ~90%). This procedure should work for any value of *P* and not just 90%.

Consider the case in which the critical and secondary sites are combined. There are 159 sites.

Figure 7. Probability of detecting at least one violation for the critical and Secondary WMD sites, *N*=159







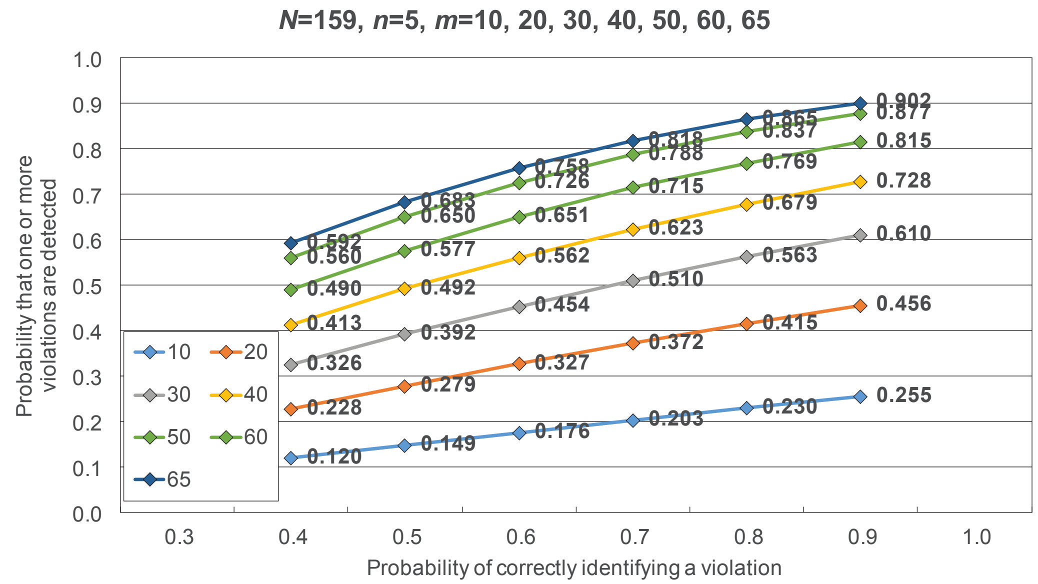


Figure 7 shows four charts each showing P with n = 2, 3, 4 and 5. The minimum numbers of inspections required are 121, 95, 77 and 65 respectively with P values between 90.2 and 90.3%. p is fixed at 0.9. One could have estimated these numbers approximately by following the procedure mentioned. For instance, when n = 2, the fixed percentage of N is ~77% with p fixed at 0.9. The minimum number of inspections required is ~77% of 159 which comes out to be ~122 sites. Similarly, ~59% of 159 gives ~94 sites with the same p when n = 3. These numbers are fairly close to the numbers observed in the charts.

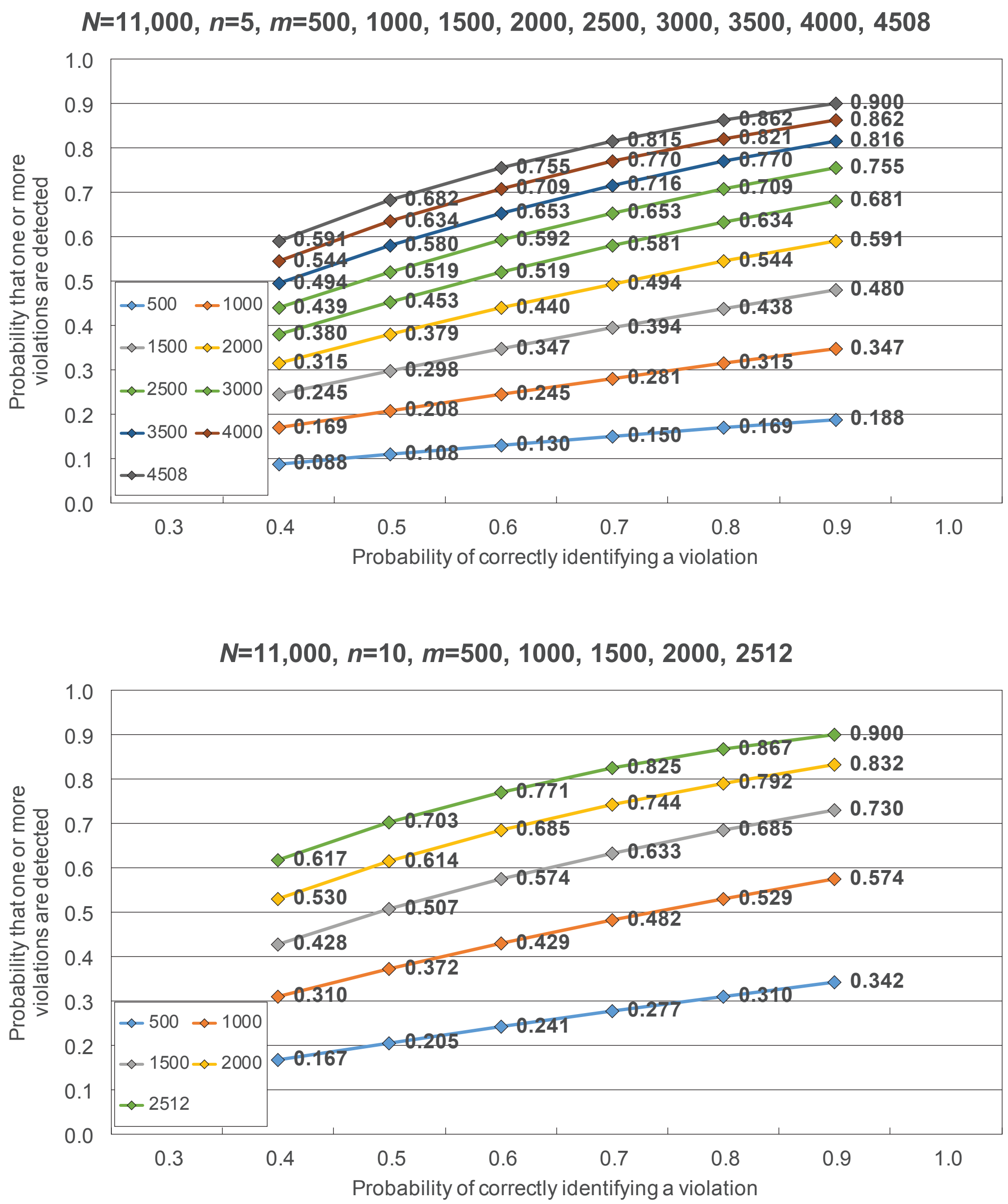
It is noted that all nuclear, chemical and biological weapons sites as well as ballistic missiles sites are treated equally in the models. In reality, however, nuclear weapons sites will take priority over chemical weapons sites due to sheer explosive power. Our assumption on random selection of m inspection sites will no longer hold in this case. It must be also borne in mind that these sites, N, will certainly be different in their size, types (e.g., above or below ground) and have specific functional purpose(s). Nevertheless, all sites are broadly generalized and treated as equal candidates for inspection in a simulation.

4.4 Underground Facilities (Joseph Bermudez)

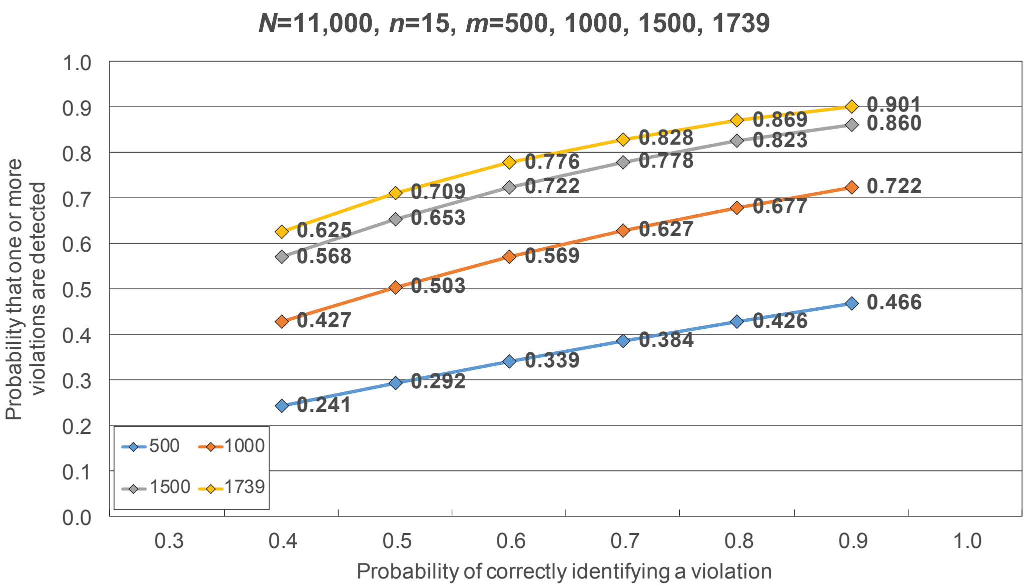
In North Korea, there are an estimated 11,000~14,000 underground facilities according to Joseph Bermudez, the author of three books on the North Korean Military. Stockpiles of biological, chemical, nuclear weapons or mobile missile launchers could be easily hidden in any of these facilities, and they could even house a production line for producing highly-enriched uranium. The number of sites, N, is of the order of thousands rather than hundreds and, hence, more inspections, m, are needed to meet the threshold. But exactly how many inspections are needed?

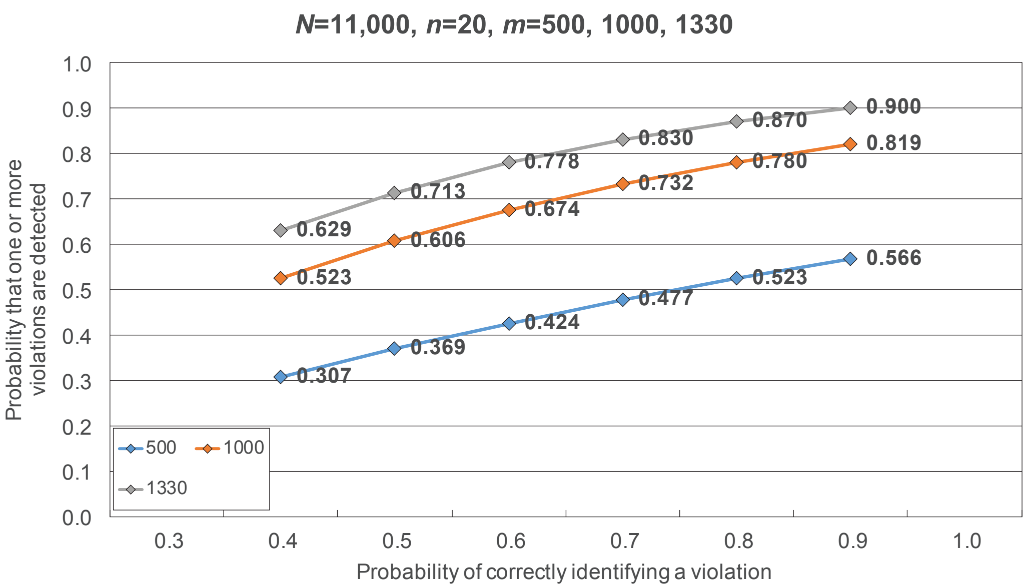
In Figure 8 below, four charts show P with n taking a value of 5, 10, 15 and 20. These values of n are extremely small relative to 11,000 potential sites of inspection. Even when n = 20, this only amounts to 0.18% of 11,000. Just to illustrate, 10~20 nuclear weapons might have been assembled in North Korea according to Kristensen and Norris.[18] In terms of relevance to our study, suppose there are 10 nuclear weapons. Then, each site in violation will, on average, hide 2 nuclear weapons when n = 5 and one nuclear weapon when n = 10. With 20 nuclear weapons, the average number increases to 4 and 2 nuclear weapons respectively for the previous cases and for n = 20, North Korea will hide one nuclear weapon in each site. Now, for n = 5, the threshold will be met if 4,508 inspections can be carried out with p fixed at 0.9. For n = 10, 15 and 20, the minimum numbers of inspections required decrease to 2,512, 1,739 and 1,330 respectively. When n = 20, P reaches 81.9% after carrying out 1,000 inspections with the same p. But when n = 5, P is only 34.7% with the same number of inspections and p. It needs additional

Figure 8. Probability of detecting at least one violation for 11,000 underground sites



18. Hans M. Kristensen & Robert S. Norris, North Korean nuclear capabilities, 2018, Bulletin of the Atomic Scientists, 74:1, 41-51, DOI: 10.1080/00963402.2017.1413062.

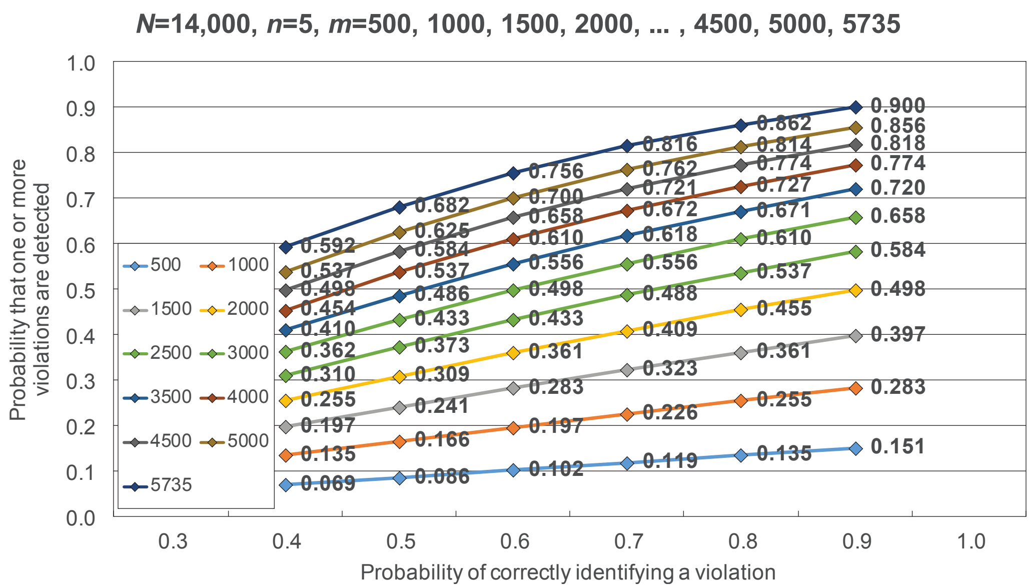


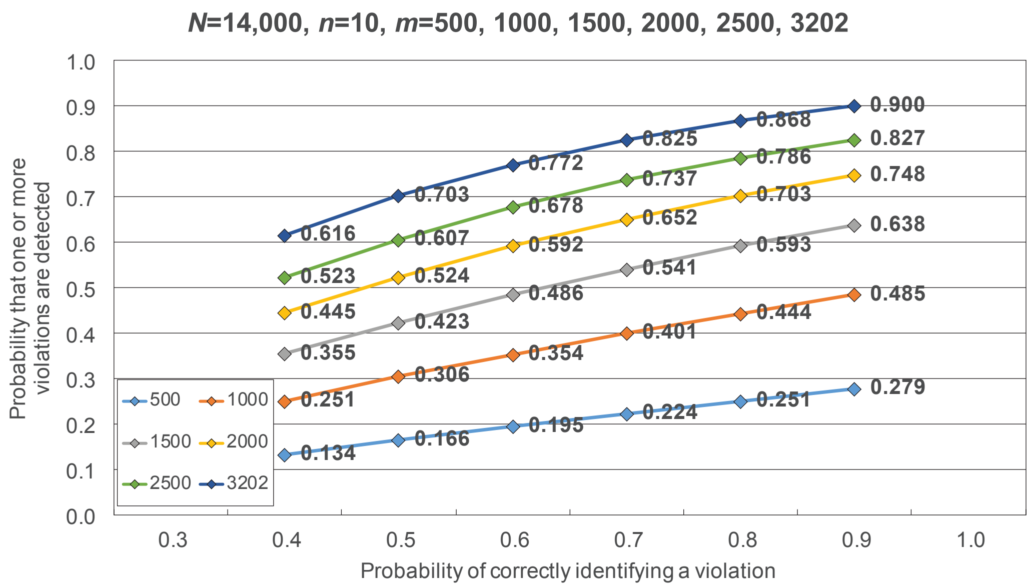


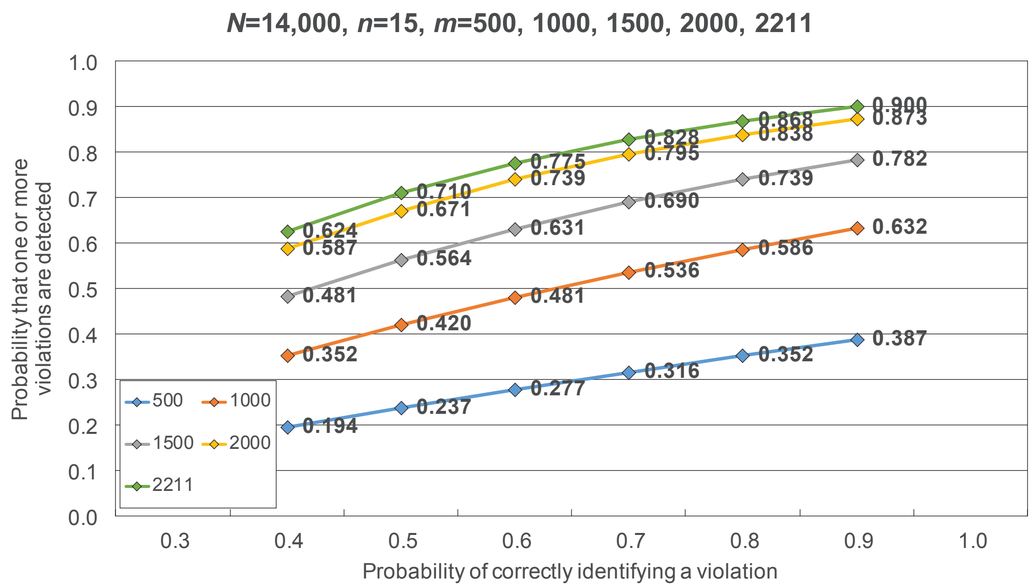
2,500 inspections for *P* to reach a similar level of 81.6%. Although not included in Figure 8, the minimum number of inspections required for $n = 2$ is estimated to be 8,356 while p is fixed at 0.9. Suppose that North Korea has used one more site to hide its nuclear weapons. So, 3 sites are now in violation (i.e., $n = 3$). The minimum number of inspections required in this case is 6,550 with the same p. This means that approximately

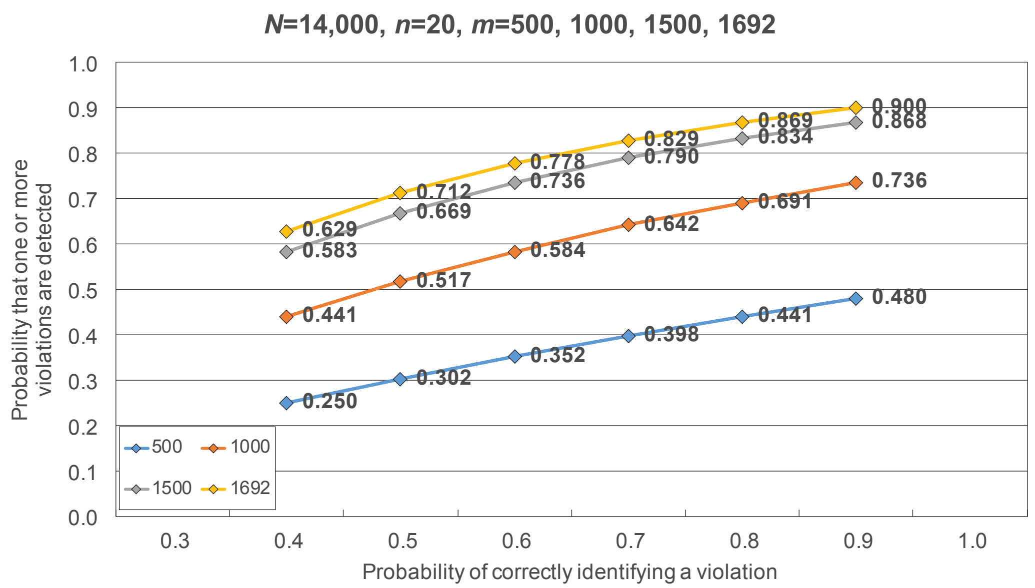
1,806 inspections can be dispensed with while the threshold is still being met. This calculation shows that one more / less site in violation can subtract / add a substantial number of inspections to meet the threshold when N is very large and n is fairly small.

Figure 9. Probability of detecting at least one violation for 14,000 underground sites









Finally, we have investigated how P evolves when there are 14,000 underground sites. Figure 9 has four charts displaying various values of P with n = 5, 10, 15 and 20.

For each n, the threshold is attained with 5,735, 3,202, 2,211 and 1,692 inspections. As numbers suggest, inspections involve searching a large number of sites. It is unprecedented

in scale and poses a serious challenge to the inspectors considering only a limited number of inspectors available. The International Atomic Energy Agency (IAEA) has 300 inspectors.[19] But although the large numbers involved in inspection appear discouraging at first sight, it might not be impossible to carry out the inspections on these sites. To a trained eye, recognizing the sites in violation from those that are not might not be too taxing. Some of these sites should be no more than a short stopover for the inspectors. However, they could encounter impediments to gaining access to these sites. This will only lengthen economic sanctions imposed on North Korea and could bring about new ones, too.

5. Quick Estimation of *m*, the number of inspections required

In this chapter, we have represented m as a percentage of N for given n, P and p. In Section 4.3, it was shown that m can be roughly estimated for a new N once these percentages are readily available. Hitherto, our focus has been on determining the minimum number of inspections required to give the threshold P value of 90% with p = 0.9. Equally, p = 0.8 or 0.7 could have been used instead. Tables 3 to 5 provide these values of m as a percentage of N. Each table has a fixed value of p ranging from 0.9~0.7.

Table 3. The number of inspections required for P as a percentage of N when p = 0.9 (N used: 27~14,000)

	n				
P	2	3	4	5	10
90% (Threshold)	76.0~77.8 %	59.3~59.5 %	48.1~48.6 %	40.7~41.0 %	22.2~22.9 %
80%	61.4~63.0 %	46.2~48.1 %	36.8~37.0 %	29.6~30.6 %	14.8~16.5 %

19. The IAEA, https://www.iaea.org/newscenter/news/new-application-increases-efficiency-and-effectiveness-of-safeguards-verification.

Table 4. The number of inspections required for P as a percentage of N when p = 0.8 (N used: 27~14,000)

	n				
P	2	3	4	5	10
90% (Threshold)	85.2~85.4 %	66.7~67.0 %	54.7~55.6 %	44.4~46.1 %	25.7~25.9 %
80%	69.1~70.4 %	51.9 %	40.7~41.4 %	33.3~34.4 %	18.5~18.6 %

Table 5. The number of inspections required for P as a percentage of N when p = 0.7 (N used: 27~14,000)

	n				
P	2	3	4	5	10
90% (Threshold)	97.7~100.0 %	76.6~77.8 %	62.6~63.0 %	51.9~52.7 %	29.4~33.3 %
80%	79.0~81.5 %	59.3 %	47.3~48.1 %	39.3~40.7 %	21.3~22.2 %

On the choice of n, we have deliberately used low values ranging from 2 to 5. N takes its minimum and maximum values of 27 and 14,000 from our simulations earlier. For given n, P and p, each cell represents (m/N) in percentage terms. When working out each of these percentages, m is the minimum number of inspections that satisfy $P \geq 90\%$ or $P \geq 80\%$ depending on the context. It is noted that as m needs to be a whole number (i.e., an integer), we have '≥' instead of '='. It does not make sense for the number of inspections to be a fractional number. This gives rise to some fluctuations observed in (m/N). Hence, a range of values is entered rather than a single value of (m/N). A small value indicates fewer inspections. Knowing the appropriate value of (m/N) for given n, P and p will immediately provide one with the minimum number of inspections required for a new N. These *'back of the envelope calculations'* are very handy in approximating m. If necessary, one can always run Monte Carlo simulations to obtain more accurate estimates.

6. Conclusion

Our research has considered the probability of detecting at least one violation assuming that North Korea is violating the law/violating its agreement guidelines. This is captured in P and its threshold is set at 90%. Then, the minimum number of inspections required to meet the threshold is determined from Monte Carlo simulations. There are also other factors that are used as input data (i.e., N, n and p). Using the tables provided in Chapter 5, one can approximately estimate the minimum number of inspections required, m, for given N, n and p. It is plainly obvious that when the number of sites in violation is small, it is inevitable to search a large number of sites to achieve a high degree of assurance. Our examples explicitly show some of the numbers involved in this regard. Undoubtedly, having unrestricted access to all sites is the way forward even though, in reality, it may prove difficult to search. It must be borne in mind that the model is trying to capture an idealized situation as it greatly simplifies the situation. Nothing in the model refers to sites which we do not know, those not included in N. To borrow a phrase from former U.S. Secretary of Defense Donald Rumsfeld, these are "known unknowns." North Korea's long running nuclear and missile programs are much more extensive than those of Iraq, Libya and Iran. Some of the facilities are likely to be in operation at secret locations. p has many factors contributing to it such as the number of inspectors, their expertise and a feeling of fatigue, to name a few. The actual value of p is determined by factors both known and unknown. Our approach has been probabilistic and based on randomly selected sites for inspection. Besides this probabilistic aspect, technical know-how and good intelligence can narrow down the possibilities and speed up the inspection process. All these different elements should come together to make effective assessment before taking the next step.

Strategically, it is in our interest to have a broad agreement in which North Korea provides a complete list of all its nuclear programs and allows the inspectors to carry out their duties without impediments. International sanctions can be gradually lifted depending on the progress made and always re-imposed if North Korea is found to be in violation or hampering inspections. Trust-building is undoubtedly an important element to bring the difficult process to a successful conclusion. For South Korea and the United States, a high level of assurance over North Korea's denuclearization is required. This necessitates North Korea rejoining the Nuclear Nonproliferation Treaty as a non-nuclear weapon state. For North Korea, it needs security assurance, the lifting of all sanctions and international recognition as a normal state.[20] Until both sides can reach a reasonable agreement, it is crucial to maintain sanctions pressure as it is the only non-military leverage at our disposal. There have been some illegal shipments of North Korean coal into South Korea via Russia. This is a clear violation of UN Security Council resolutions. South Korea's borders must be tightened and any suspicious shipment thoroughly investigated to avoid repeating similar incidents in future. It will be very difficult for South Korea to ask China or Russia to stringently abide by its international obligations when it itself has failed and when it keeps on asking for exemptions. Such an incident undermines its own credibility. South Korea's interest in pursuing economic collaboration with North Korea must be in step with the degree of progress made on denuclearization. Its actions must be consistent, in line with its allies. To that effect, the U.S. and South Korea must consult each other more closely to coordinate their strategy towards North Korea.[21] At times, the two countries appear to be out of sync, which can only undermine each other's confidence. The U.S. must shoulder the primary responsibility for maintaining the credibility and effectiveness of its extended deterrence to South Korea as long as the North Korean threat remains. At no time in the process should the defense and security of South Korea be compromised on the basis of North Korea's goodwill and good intentions. South Korea must never let its guard down of its own accord especially when North Korea remains an illegal nuclear holder.[22] South Korea's strategy must not be based on unrealistic expectations and wishful thinking. North Korea's nuclear and missile programs pose a direct existential threat to the South Korea's national security, to say nothing of their consequences beyond its borders. Much effort is needed towards reaching the kind of broad agreement referred above. Given the circumstances, it is a preferred and pragmatic way of resolving North Korea's nuclear issue from our perspective.

However, from North Korea's perspective, it may believe that its interest lie in inhibiting inspections of its nuclear sites even though such actions will prolong economic hardship. North Korea may think that by pursuing salami tactics, it can eventually reach a full compensation for any economic hardship it has endured along the way. The benefits North Korea would obtain through aggregate compensation are expected to be much more than it would, otherwise, have received in return for simply letting the inspectors in and cooperating with the international community. North Korea's asymmetric strategy can be adopted here to maximize its gains in exchange for a razor-thin slice of salami. North Korea can prolong the nuclear deadlock while implementing minimum measures which fall far short of actual dismantlement. In the meantime, it can pay lip service to its intentions towards denuclearization. If we are forced to face this much less-preferred scenario in reality, then our response should be *'commensurate'* with what North Korea is willing to give up. It is, therefore, necessary for us to have our own selection of razor-thin salami slices, enough in numbers, to match those of North Korea's.[23] To this end, South Korea and its allies must work together closely to identify their own types of responses and how they can be used at each stage of the negotiation. The list of responses will include sectors such as economic, political and military. It is very important to set up a reasonable timeline as this process cannot continue forever. Surely it will not take long to know North Korea's true intentions. Furthermore, additional measures must be established as North Korea can always reverse its position and return to its former volatile self by threatening to launch missiles and increase its production of nuclear weapons and ballistic missile delivery systems. Falling into the trap of entering this type of *'slicing ever thinner salami'* competition is best avoided. North Korea must be convinced that there is nothing to be gained from pursuing this path rather than the one under the broad agreement. North Korea's propensity for salami slicing must be thwarted from the outset with appropriate responses.

20. It will be very difficult to be internationally recognized as a normal state without addressing its poor human rights record.
21. Establishing a new joint working group will facilitate coordination between the two countries.
22. South Korea should have been more prudent before signing the joint statement in Pyongyang.

23. For example, it is better to scale down or reduce the frequency of a joint military exercise rather than stopping it altogether. The minimum level of readiness must be maintained at all times. North Korea still has massive conventional forces, and there are potential threats from other states in the region.

Appendix – Inspection Violation (Confidence) Model

A1. An Analytical Technique

Suppose that there are altogether N detected disturbances[24] among which n are violations and suppose $m \le n$ inspections are allowed. Let p be the probability of correctly identifying a violation. Then, P denotes the probability that M or more violations are detected, given by the equation below.[25]

$$P(M,N,n,m,p) = \sum_{i=M}^{m}\left[\binom{n}{i}\binom{N-n}{m-i}/\binom{N}{m}\right]\sum_{j=M}^{i}\left[\binom{i}{j}p^{j}(1-p)^{i-j}\right]$$

The mathematical terms in the first summation work out the number of ways in which violating and non-violating sites can be chosen in a sample (i.e., m). They are recognized as the probability mass function of the hyper-geometric distribution. In the meanwhile, the terms in the second summation describe how successfully the inspectors can identify a violating site to be indeed violating. They represent the familiar binomial probability formula. Here, $\binom{n}{i}$ denotes a binomial coefficient. For the equation to be valid, the following three constraints should hold. First, $M \le m$ as one cannot find more violation(s) than the number of inspections allowed. Second, $n \le N$ as n is simply taken to be violations among N. Third, $m \le n$ as $\binom{n}{i}$ is mathematically undefined for the case $m > n$.[26]

It is also assumed that there is at least one violation (i.e., $n \ge 1$). The case in which there is no violation (i.e., $n=0$) is not of much interest. Also, when considering the lower limit for M, the issue that is of critical importance is whether there is actually a violation or not. The actual number of violations is of secondary importance. Hence, the lower limit

24. This can be simply the number of known nuclear sites such as the number of buildings or facilities rather than the number of disturbances detected. This would depend on the context.

25. Thomas Saaty, "Mathematical Models of Arms Control and Disarmament." Application of Mathematical Structures in Politics, John Wiley & Sons, Inc, 1968.

26. One can recognize this third constraint as the limitation of the equation P. This is rather restrictive.

for M is set to 1. P is now defined to be the probability of detecting one or more violations, and this definition is used throughout our research. There should be no confusion between P and p, as the latter directly represents such measure as how skilled and competent the inspectors are in detecting violation once they are at a violating site.

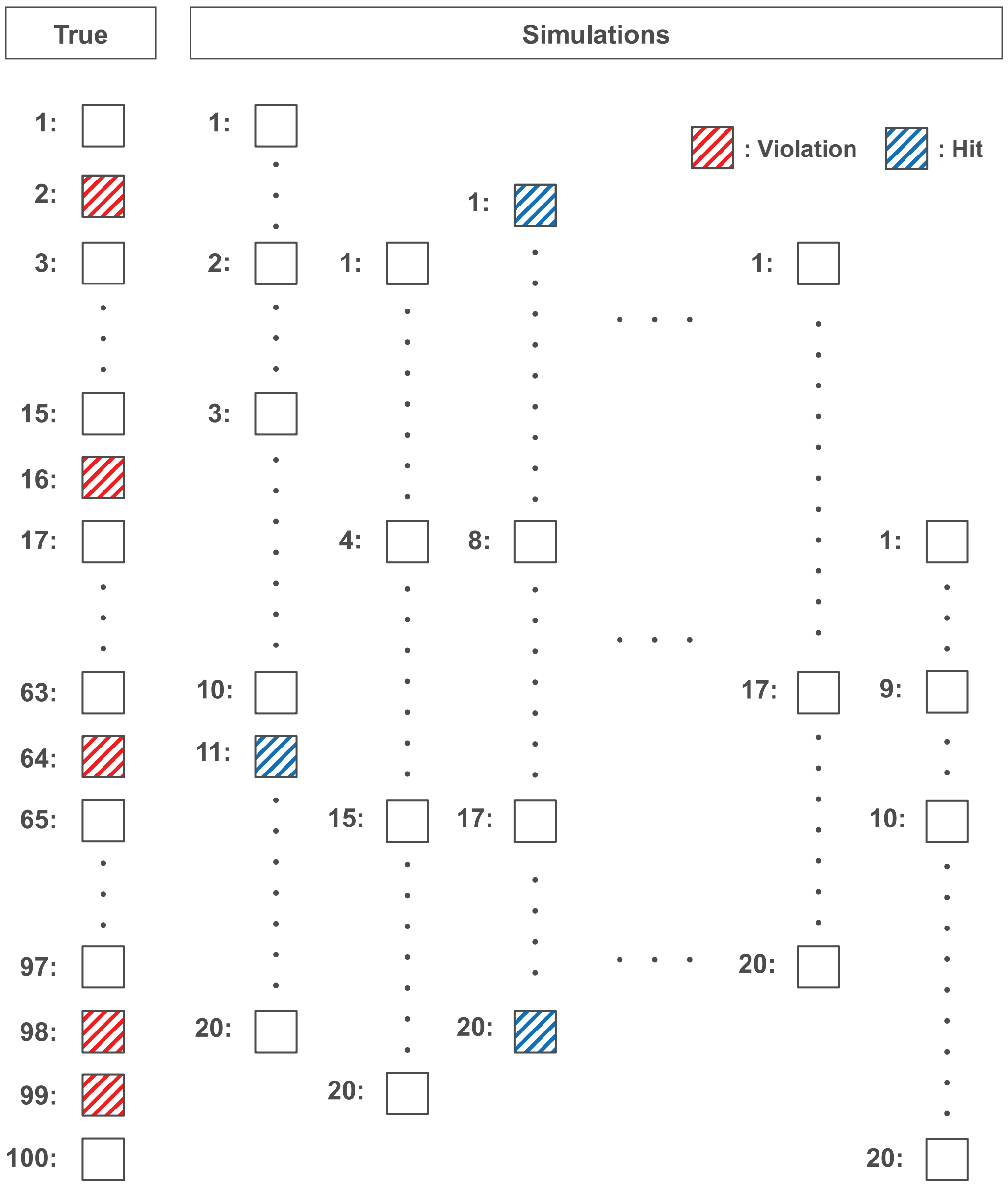
A2. A Monte Carlo Technique

In this section, we explain in detail how Monte Carlo Technique is used to determine P. Figure 10 below illustrates Monte Carlo simulations with 1,000 iterations. It has been found that 1,000 iterations provide sufficient accuracy for our purpose, although a general rule of thumb is to use 10,000 iterations. A column of 100 squares on the LHS figuratively represents 100 nuclear sites (i.e., N = 100) each numbered from 1 to 100. Suppose that North Korea has hidden its nuclear weapons in 5 sites (i.e., n = 5) among the 100 sites as indicated by the red squares. So, there are 5 sites in violation. The number of inspections allowed is assumed to be 20 (i.e., m = 20). The second column shows 20 randomly chosen sites each numbered from 1 to 20.[27] This is the first iteration. The first inspection site chosen in this iteration is not a site in violation and so is the second site chosen. Only the 11th chosen site happens to be a site in violation. This corresponds to the 64th site on the LHS column which is one of the 5 sites in violation.

However, choosing the 64th site does not necessarily guarantee that the inspectors have caught North Korea in violation. The inspectors must carry out their tasks diligently to find incriminating evidence. Their chances of success are given by probability p and failure by 1 - p. Figure 11 below shows this situation. In this particular instance, they have been successful and the blue square encircled in a dashed blue circle represents this fact. It could have gone the other way ending up with a white square rather the blue one. This is exactly what has happened to the last (i.e., 20th) inspection site chosen as indicated by the white square. It has failed to detect a violation at the 98th site. This completes the first iteration, and there is only one hit recorded. Simulation re-starts with the second iteration and continues all the way up to the thousandth iteration. The number of hits (i.e., the number of violations detected) is recorded at the end of each iteration. The number of hits ranges from 0 to 5 as there can only be 5 sites in violation at maximum (i.e., 5 red squares). The numbers are recorded in the first row of the table

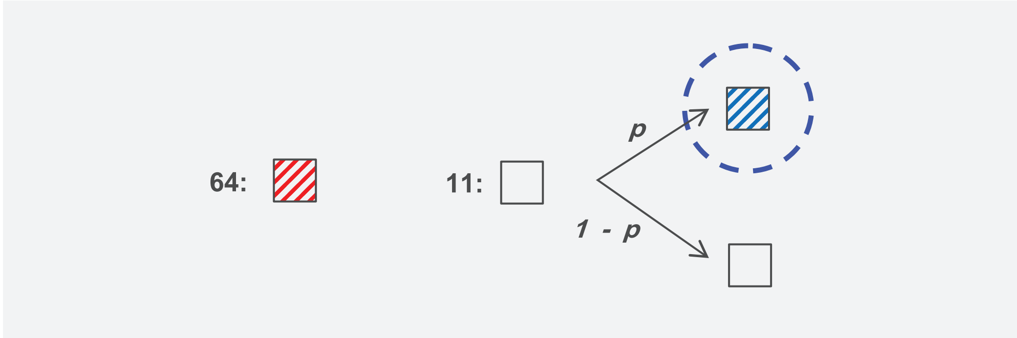
27. It is ensured during simulation that each of the 20 sites is chosen only once.

Figure 10. A Monte Carlo Technique with 1,000 iterations



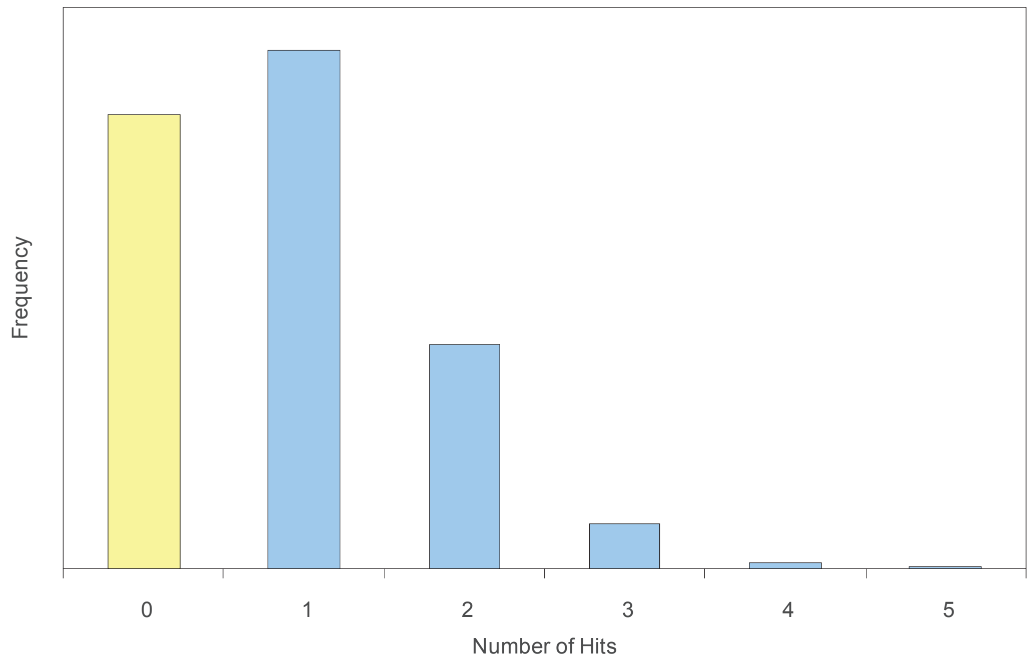
Number of Hits	1	0	2	···	0	0
Iteration Number	1	2	3	···	999	1,000

Figure 11. Detection success with probability p and failure with probability $1 - p$



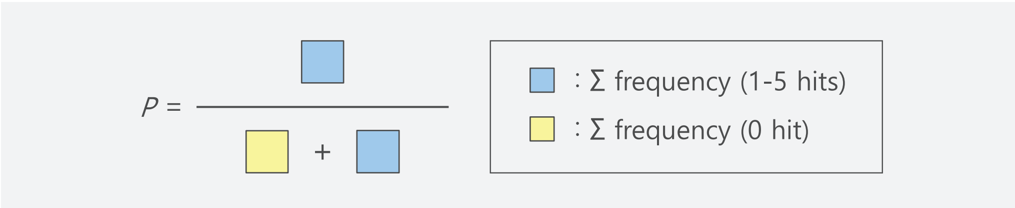
underneath Figure 10. Now, we are interested in determining P. For this, we need to count how many 0, 1, 2, 3, 4 and 5 hits there are each among the 1,000 iterations. Figure 12 shows the frequency distribution graph. The x and y-axes denote the number of hits and the frequency respectively.

Figure 12. Frequency distribution



P is defined to be the probability of detecting at least one violation assuming that North Korea is cheating. So, dividing the sum of frequencies for 1 to 5 hits by the sum of all frequencies will give the value of *P*. Figure 13 shows this pictorially.

Figure 13. Determining *P* using a Monte Carlo Technique



For this particular example, *P* is 0.636 (i.e., 63.6%) when *p* is fixed at 0.9.

ASAN REPORT

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